An Open Economy New-Keynesian Model

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Outline

- Review of the past
- New-Keynesian economics
  - Basic model open economy model
    - Household’s problem
    - Firms’ problem
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      - Intermediate good producers
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  - Extensions
    - Backward-looking element in firms’ problem
    - Sticky wages
    - Households’ habit formation

Review of the past

- Lucas and Sargent (1978) launched rational expectations revolution:
  - *"…the predictions [of Keynesian economics] were wildly incorrect, and that the doctrine on which they were based was fundamentally flawed…"*
- In the last few decades, two groups of business cycle economists emerged:
  - Neo-classicals
  - New-Keynesians

Neo-classicals (fresh water, RBC)

- Embraced the Lucas-Sargent call for reconstruction
- Nominal rigidities, imperfect information, money, and the Phillips curve disappeared from the basic model.
- Three principles guided this approach:
  - Explicit micro foundations
    - Utility and profit maximization
  - General equilibrium
  - The exploration of how far one could go with no or few imperfections

New-Keynesian (salt water)

- Embraced reform, not revolution
- They examine the nature and the reality of various imperfections:
  - Nominal rigidities,
  - Efficiency wages,
  - Credit market constraints.
- Less emphasis on general equilibrium compared to Neo-classicals

Neo-classicals vs. New-Keynesians

- NC accused NK of being "bad" economists, clinging to obsolete beliefs and discredited theories
- NK accused the NC of ignoring basic facts in their pursuit of a beautiful but irrelevant model

In the end…

- The basic NK model starts from the RBC model and adds two main imperfections:
- Monopolistic competition in the goods market.
  - If the economy is going to have price setters, they better have some monopoly power.
- Discrete nominal price setting (sticky or staggered prices and/or wages).

Convergence in methodology

- Dynamic Stochastic General Equilibrium models (DSGEs).
- Derived from micro foundations
  - Utility maximization by consumers-workers, profit maximization by firm
  - Rational expectations
  - Specification of imperfections
  - Nominal rigidities
- Can be estimated using data possibly with Bayesian methods
- Nearly every central bank has one or wants to have one!

Before we start: Terminology

- Structural: Each equation has an economic interpretation
- General Equilibrium: Demand=Supply
- Stochastic: There are random shocks
- Rational expectations: Agents use past information and the knowledge about how the economy works (the model) to make inference about future
- Dynamic: We care not only about today but also about yesterday and tomorrow

Household's problem

- Maximize:

Elasticity of substitution between today and future

$$E_t \sum_{i=0}^{\infty} \beta^{t+i} \left[\frac{C_{t+i}^{1-\sigma}}{1-\sigma} + \frac{\gamma}{1-b} \left(\frac{M_{t+i}}{P_{t+i}} \right)^{1-b} - \chi \frac{N_{t+i}^{1+\eta}}{1+\eta} \right]$$

Weight put on leisure in comparison to consumption

- Notation:
- C :consumption
- N :labor, $1-N$:leisure
- M/P :real money balances
- Consumption goods are produced by consumption good producers – bundle imports and domestic intermediate goods

Household's problem

$$E_t \sum_{i=0}^{\infty} \left[\frac{C_{t+i}^{1-\sigma}}{1-\sigma} + \frac{\gamma}{1-b} \left(\frac{M_{t+i}}{P_{t+i}} \right)^{1-b} - \chi \frac{N_{t+i}^{1+\eta}}{1+\eta} \right]$$

- Representative households with preferences defined over consumption, real money balances and leisure.
- Money in the utility:
  - Households are better off when they hold more real money balances
- Expectations are rational

Household's problem

$$C_t = \left[\int_0^1 c_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}}, \theta > 1$$

- C is a basket
- Consisting of differentiated products produced by monopolistically competitive consumption goods producers.
- There is a continuum of firms and each firm j produces good c_j
- θ is the price elasticity of demand for the individual goods.

Household’s problem

- Household’s problem can be solved in two stages
  1- Minimize cost for a given C
  2- Given the cost of achieving any given C, choose C, N and M

Household's problem: first stage

- Minimize

$$\min_{c_{jt}} \int_0^1 p_{jt} c_{jt} dj$$

subject to

$$\left[\int_0^1 c_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \geq C_t$$

where p_j is the price of good j.

Household's problem: first stage

- The Lagrange function is as follows

$$\Gamma = \int_0^1 p_{jt} c_{jt} dj + \psi_t \left[C_t - \left[\int_0^1 c_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \right]$$

Lagrange multiplier

- And the first order condition for any c_{jt}

$$\partial\Gamma \big/ \partial c_{jt} \equiv p_{jt} - \psi_t \left[\int_0^1 c_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{1}{\theta-1}} c_{jt}^{-\frac{1}{\theta}} = 0$$

Household's problem: first stage

- Using the definition of the composite good

$$C_t = \left[\int_0^1 c_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}}, \theta > 1$$

- We get

$$p_{jt} - \psi_t \left[\int_0^1 c_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{1}{\theta-1}} c_{jt}^{-\frac{1}{\theta}} = 0$$

First order condition (f.o.c.)

$$p_{jt} - \psi_t C_t^{\frac{1}{\theta}} c_{jt}^{-\frac{1}{\theta}} = 0$$

Definition of the composite good substituted in the f.o.c

$$p_{jt}^{-\theta} - \psi_t^{-\theta} C_t^{-1} c_{jt} = 0$$

Household's problem: first stage

- And finally

$$c_{jt} = (p_{jt} / \psi_t)^{-\theta} C_t$$

- We substitute this back in the definition of the composite good:

$$C_t = \left[\int_0^1 \left[\left(\frac{p_{jt}}{\psi_t} \right)^{-\theta} C_t \right]^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} = \left(\frac{1}{\psi_t} \right)^{-\theta} \left[\int_0^1 p_{jt}^{1-\theta} dj \right]^{\frac{\theta}{\theta-1}} C_t$$

Household's problem: first stage

- And solve for ψ_t :

$$\psi_t = \left[\int_0^1 p_{jt}^{1-\theta} dj \right]^{\frac{1}{\theta-1}} = P_t$$

- The Lagrange multiplier is the appropriate price index (shadow price) for consumption.
- The demand for j-th good is:

$$c_{jt} = \left(\frac{p_{jt}}{P_t} \right)^{-\theta} C_t$$

Household's problem: first stage

$$c_{jt} = \left(\frac{p_{jt}}{P_t} \right)^{-\theta} C_t$$

- This is the individual "demand curve"
- Demand for a particular good j depends on its price relative to composite good price index
- Price elasticity of demand for good j is also important. As $\theta \to \infty$, individual goods become closer substitutes and firms have less market power

Household's problem in an open economy

$$\max_{C,M,(1-N)} E_t \sum_{i=0}^{\infty} \left[\frac{C_{t+i}^{1-\sigma}}{1-\sigma} + \frac{\gamma}{1-b}\left(\frac{M_{t+i}}{P_{t+i}}\right)^{1-b} - \chi \frac{N_{t+i}^{1+\eta}}{1+\eta} \right]$$

$$\text{s.t.} C_t + \frac{M_t}{P_t} + \frac{B_t}{P_t} + \frac{S_t B_t^*}{P_t} = \left(\frac{W_t}{P_t}\right) N_t + \frac{M_{t-1}}{P_t} + (1+i_{t-1})\left(\frac{B_{t-1}}{P_t}\right) + (1+i_{t-1}^*)\left(\frac{S_{t-1}B_{t-1}^*}{P_t}\right) + \Pi_t$$

- B_t is household's nominal holdings of one-period bonds
- Bonds pay a nominal interest rate of i_t
- Real profits received from firms are Π_t
- W_t nominal wages
- B_t^* is household's nominal holdings of one-period international bonds
- Which pay a nominal interest rate of i_t^*
- While change in the exchange rate $S_t - S_{t-1}$ is important

Household's problem: second stage

- Choose consumption, labor supply, money, and bond holdings. First order conditions are:

$$C_t^{-\sigma} = \beta(1+i_t)E_t\left(\frac{P_t}{P_{t+1}}\right)C_{t+1}^{-\sigma}$$

$$\frac{\gamma\left(\frac{M_t}{P_t}\right)^{-b}}{C_t^{-\sigma}} = \left(\frac{i_t}{1+i_t}\right)$$

$$\frac{\chi N_t^{\eta}}{C_t^{-\sigma}} = \left(\frac{W_t}{P_t}\right)$$

Open economy

- First order conditions in respect to B_t and B_t^* are

$$C_t^{-\sigma} = \beta(1+i_t)E_t\left(\frac{P_t}{P_{t+1}}\right)C_{t+1}^{-\sigma}$$

$$C_t^{-\sigma} = \beta(1+i_t^*)E_t\left(\frac{S_t}{S_{t+1}}\right)E_t\left(\frac{P_t}{P_{t+1}}\right)C_{t+1}^{-\sigma}$$

- Which can be combined to obtain …

$$E_t\left(\frac{S_t}{S_{t+1}}\right) = \frac{(1+i_t^*)}{(1+i_t)}$$

- … the UIP – uncovered interest rate parity

Household's problem: second stage

$$C_t^{-\sigma} = \beta(1+i_t)E_t\left(\frac{P_t}{P_{t+1}}\right)C_{t+1}^{-\sigma}$$

- Euler equation (inter-temporal) for the optimal intertemporal allocation of consumption:
  - On the LHS: marginal utility of 1 unit of consumption today
  - On the RHS: Expected marginal utility of consumption tomorrow if decide to save that 1 unit of consumption today
  - In equilibrium these two need to be equal

Household's problem: second stage

$$\frac{\gamma\left(\dfrac{M_t}{P_t}\right)^{-b}}{C_t^{-\sigma}} = \left(\frac{i_t}{1+i_t}\right)$$

- Intratemporal optimality condition to pin down money holdings
  - On the LHS: marginal rate of substitution between money and consumption
  - On the RHS: opportunity cost of holding money

Household's problem: second stage

$$\frac{\chi N_t^{\eta}}{C_t^{-\sigma}} = \left(\frac{W_t}{P_t}\right)$$

- Intratemporal optimality condition to determine labor supply
  - On the LHS: Marginal rate of substitution between leisure and consumption
  - On the RHS: Real wage

Firms' problem – consumption goods producers

- Maximize profits subject to:
  - 1- Constant returns to scale technology

$$c_{jt} = \lambda^c Y_{jt}^C + (1-\lambda^c) M_{jt}^C$$

    - Where λ^c is share of domestic intermediate good used for production of consumption good j
  - 2- Demand curve of the HH
  - 3- Sticky prices a la Calvo (1983): explained in next slide…
- Firms are identical except that they might have set their prices at different dates in the past.

Firms' problem

- Calvo pricing
- Each period, firms that adjust their price are randomly selected
  - A fraction $1-\omega$ of firms adjust while remaining ω do not.
  - ω measures the degree of nominal rigidity
  - Higher ω $\Rightarrow$ fewer firms adjust each period, expected time between price adjustments is longer
  - Firms that adjust do so to maximize expected discounted value of current and future profits

Sticky prices

- Why should prices adjust slowly?
- One common explanation is "menu costs": small costs that must be paid in order to adjust nominal prices.
  - The costs of making a new catalog, price list, or menu.

Sticky prices

- There are also externalities that go along with changing prices:
  - A firm that lowers its prices because of a decrease in the money supply will be raising the real income of the customers of that product.
  - This will allow the buyers to purchase more, which will not necessarily be from the firm that lowered their prices.
  - As firms do not receive the full benefit from reducing their prices their incentive to adjust prices in response to macroeconomic events is reduced.

Monopolistic competition

- Without some monopoly power it doesn’t make sense to assume sticky prices!
  - Under perfect competition, any firm with a price slightly higher than the others would be unable to sell anything. Any firm with a price slightly lower than the others would be obliged to sell much more than they can profitably produce.
- Firms use their market power to maintain their prices above marginal cost, so that even if they fail to set prices optimally they will remain profitable.

Consumption goods producers: First stage

- Cost minimization: minimize cost of production subject to producing a given amount

$$\min_{Y_t, M_t} \frac{\left(P_t^Y Y_{jt}^C + P_t^M M_{jt}^C\right)}{P_t} + \varphi_t \left(c_{jt} - \lambda^c Y_{jt}^C - (1-\lambda^c) M_{jt}^C\right)$$

- φ_t is the firms' marginal cost.
- First order conditions are:

$$\varphi_t = \frac{P_t^Y}{\lambda^c P_t}$$

$$\varphi_t = \frac{P_t^M}{\left(1-\lambda^c\right) P_t}$$

Consumption goods producers: Second stage

- Price setting: choose price to maximize present discounted value of profits

Cost of production in real terms

$$\max_{p_{jt}} \Pi_t = E_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+i} \left[\left(\frac{p_{jt}}{P_{t+i}} \right) c_{jt+i} - \varphi_{t+i} c_{jt+i} \right]$$

Probability of not-being able to change the price

Revenues in real terms

- Where $\Delta_{i,t+i} = \beta^i \left(C_{t+i} / C_t \right)^{-\sigma}$ is the discount factor
- It is the same as households' discount factor since households are assumed to own the firms

Consumption goods producers: Second stage

- Substituting households' demand curve for the j-th firm production in the j-th firm's profit maximization:

$$\max_{p_{jt}} E_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+i} \left[\left(\frac{p_{jt}}{P_{t+i}} \right)^{1-\theta} - \varphi_{t+i} \left(\frac{p_{jt}}{P_{t+i}} \right)^{-\theta} \right] C_{t+i}$$

- Let p_t^* be the optimal price chosen by firms adjusting at time t. Then the firm's f.o.c. is:

$$\partial \Pi_t \Big/ \partial p_t^* \equiv E_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+i} \left[(1-\theta) \left(\frac{p_t^*}{P_{t+i}} \right) + \theta \varphi_{t+i} \right] \left(\frac{1}{p_t^*} \right) \left(\frac{p_t^*}{P_{t+i}} \right)^{-\theta} C_{t+i} = 0$$

Consumption goods producers: Second stage

- The f.o.c. is solved for p_t^* :

$$p_t^* = \left(\frac{\theta}{\theta-1}\right)\frac{E_t\sum_{i=0}^{\infty}\omega^i\Delta_{t,t+i}\varphi_{t+i}P_{t+i}^{\theta}C_{t+i}}{E_t\sum_{i=0}^{\infty}\omega^i\Delta_{t,t+i}P_{t+i}^{\theta-1}C_{t+i}}$$

- Using $\Delta_{i,t+i} = \beta^i\left(C_{t+i}/C_t\right)^{-\sigma}$ it can be rearranged in real terms as:

$$\left(\frac{p_t^*}{P_t}\right) = \left(\frac{\theta}{\theta-1}\right)\frac{E_t\sum_{i=0}^{\infty}\omega^i\beta^iC_{t+i}^{1-\sigma}\varphi_{t+i}\left(\frac{P_{t+i}}{P_t}\right)^{\theta}}{E_t\sum_{i=0}^{\infty}\omega^i\beta^iC_{t+i}^{1-\sigma}\left(\frac{P_{t+i}}{P_t}\right)^{\theta-1}}$$

Consumption goods producers: Second stage

- If all firms could adjust every period, i.e. under flexible prices: $(\omega = 0)$

$$\left(\frac{p_t^*}{P_t}\right) = \left(\frac{\theta}{\theta - 1}\right)\varphi_t = \mu\varphi_t$$

  - Each firm sets its price to a markup μ over its nominal marginal cost $P_t\varphi_t$
  - A standard result in a model of monopolistic competition
  - As price>marginal cost, output is inefficiently low even under flexible prices!

- Further it holds that under flexible prices all firms set the same price, $p_t^* = P_t$ and $\varphi_t = 1/\mu$

Consumption goods producers: Second stage

- Remember:

$$\psi_t = \left[\int_0^1 p_{jt}^{1-\theta} dj \right]^{\frac{1}{\theta-1}} = P_t$$

- Since the adjusting firms are chosen randomly form a continuum of firms following aggregation holds:

$$P_t^{1-\theta} = (1-\omega)\left(p_t^*\right)^{1-\theta} + \omega P_{t-1}^{1-\theta}$$

- That is in fact what is so attractive on Calvo pricing.

Intermediate goods producers

- Maximize profits subject to:
  - 1- Constant returns to scale technology

$$y_{jt} = Z_t N_{jt}, E(Z_t) = 1$$

    - Where Z is an aggregate productivity shock
  - 2- Demand of consumption goods producers and exporters
  - 3- Sticky prices a la Calvo (1983)
- What's different is the production function, rest is same!

Intermediate goods producers: First stage

- Cost minimization: minimize cost of production subject to producing a given amount

$$\min_{N_t} \left(\frac{W_t}{P_t^Y} \right) N_t + \varphi_t^y \left(y_{jt} - Z_t N_{jt} \right)$$

- φ_t^y is the marginal cost of intermediate producers.
- First order condition is:

$$\varphi_t^y = \frac{W_t / P_t^Y}{Z_t}$$

Intermediate producers: Second stage

- It ends with:

$$\left(\frac{p_t^{y*}}{P_t^Y}\right) = \left(\frac{\theta}{\theta - 1}\right) \frac{E_t \sum_{i=0}^{\infty} \omega_y^i \beta^i C_{t+i}^{1-\sigma} \varphi_{t+1}^y \left(\frac{P_{t+i}^Y}{P_t^Y}\right)^{\theta}}{E_t \sum_{i=0}^{\infty} \omega_y^i \beta^i C_{t+i}^{1-\sigma} \left(\frac{P_{t+i}^Y}{P_t^Y}\right)^{\theta-1}}$$

- And

$$\left(P_t^Y\right)^{1-\theta} = \left(1 - \omega_y\right)\left(p_t^{Y*}\right)^{1-\theta} + \omega_y \left(P_{t-1}^Y\right)^{1-\theta}$$

Importers

- Maximize profits subject to:
  - 1- Constant returns to scale technology

$$m_{jt} = M^*_{jt}$$

    - Importers simply import
  - 2- Demand curve of consumer goods producers and exporters
  - 3- Sticky prices a la Calvo (1983
  - Firms are identical except that they might have set their prices at different dates in the past.

Importers: First stage

- Cost minimization: minimize cost of production subject to producing a given amount

$$\min_{M^*_{jt}} \frac{S_t P_t^*}{P_t^M} M^*_{jt} + \varphi_t^m \left(m_{jt} - M^*_{jt} \right)$$

Production is not a constraint here

- φ_t^m is the marginal cost of importers
- There isn’t much sense in the minimization
- It is clear that marginal cost on any additional unit of import is $S_t P_t^*$
- An real marginal cost is given by $\varphi_t^m = \frac{S_t P_t^*}{P_t^M}$

Importers: Second stage

- It ends with:

$$\left(\frac{p_t^{m*}}{P_t^M}\right)=\left(\frac{\theta}{\theta-1}\right)\frac{E_t\sum_{i=0}^{\infty}\omega_m^i\beta^i C_{t+i}^{1-\sigma}\varphi_{t+1}^m\left(\frac{P_{t+i}^M}{P_t^M}\right)^{\theta}}{E_t\sum_{i=0}^{\infty}\omega_m^i\beta^i C_{t+i}^{1-\sigma}\left(\frac{P_{t+i}^M}{P_t^M}\right)^{\theta-1}}$$

- And

$$\left(P_t^M\right)^{1-\theta}=\left(1-\omega_y\right)\left(p_t^{m*}\right)^{1-\theta}+\omega_y\left(P_{t-1}^M\right)^{1-\theta}$$

Exporters

- Maximize profits subject to:
  - 1- Constant returns to scale technology

$$x_{jt} = \lambda^x Y_{jt}^X + (1-\lambda^x) M_{jt}^X$$

    - Where λ^x is share of domestic intermediate good used for production of consumption good j
  - 2- Foreign demand
  - 3- Sticky prices a la Calvo (1983)
- Firms are identical except that they might have set their prices at different dates in the past.

Exporters: First stage

- Cost minimization: minimize cost of production subject to producing a given amount

$$\min_{Y_t, M_t} \frac{\left(P_t^Y Y_{jt}^X + P_t^M M_{jt}^X\right)}{P_t^X} + \varphi_t^x \left(x_{jt} - \lambda^x Y_{jt}^X - (1-\lambda^x) M_{jt}^X\right)$$

- φ_t is the firms' marginal cost.
- First order conditions are:

$$\varphi_t^x = \frac{P_t^Y}{\lambda^x P_t^X}$$

$$\varphi_t^x = \frac{P_t^M}{\left(1-\lambda^x\right) P_t^X}$$

Exporters: Second stage

- It ends with:

$$\left(\frac{p_t^{x*}}{P_t^X}\right)=\left(\frac{\theta}{\theta-1}\right)\frac{E_t\sum_{i=0}^{\infty}\omega_x^i\beta^iC_{t+i}^{1-\sigma}\varphi_{t+1}^x\left(\frac{P_{t+i}^X}{P_t^X}\right)^{\theta}}{E_t\sum_{i=0}^{\infty}\omega_x^i\beta^iC_{t+i}^{1-\sigma}\left(\frac{P_{t+i}^X}{P_t^X}\right)^{\theta-1}}$$

- And

$$\left(P_t^X\right)^{1-\theta}=\left(1-\omega_x\right)\left(p_t^{x*}\right)^{1-\theta}+\omega_x\left(P_{t-1}^X\right)^{1-\theta}$$

Quantities

- Consumption is determined by households
- Consumption goods producers demand intermediate goods and imports

$$Y_t^C = \lambda^c \left(\frac{P_t}{P_t^Y} \right)^{\theta} C_t$$

$$M_t^C = \left(1 - \lambda^c\right) \left(\frac{P_t}{P_t^M} \right)^{\theta} C_t$$

Quantities

- Exporters also demand intermediate goods and imports

$$Y_t^X = \lambda^x \left(\frac{P_t^X}{P_t^Y} \right)^\theta X_t$$

$$M_t^X = \left(1 - \lambda^x\right) \left(\frac{P_t^X}{P_t^M} \right)^\theta X_t$$

Quantities

- While the volume of exports is determined by foreign demand and the real exchange rate

$$X_t = \left(\frac{P_t^X S_t}{P_t^*} \right)^{\vartheta} M_t^*$$

Current account, net external position, risk premium and UIP

$$B_t = B_{t-1}\left(1+i_t^*\right)\left(1+\Delta S_t\right)+P_t^X X_t - P_t^M M_t$$

$$b_t = b_{t-1}\left(1+i_t^*\right)\frac{P_{t-1}^X X_{t-1}}{P_t^X X_t}+1-\frac{P_t^M M_t}{P_t^X X_t}$$

$$prem_t = -\xi b_t$$

$$s_{t+1}-s_t = i_t - i_t^* - prem_t$$

Net foreign assets divided by exports

Log version of the UIP extended for the risk premium

Sum up … aggregated version

- Households

$$C_t^{-\sigma} = \beta(1+i_t)E_t\left(\frac{P_t}{P_{t+1}}\right)C_{t+1}^{-\sigma}$$

$$\frac{\chi N_t^{\eta}}{C_t^{-\sigma}} = \left(\frac{W_t}{P_t}\right)$$

$$\frac{\gamma\left(\frac{M_t}{P_t}\right)^{-b}}{C_t^{-\sigma}} = \left(\frac{i_t}{1+i_t}\right)$$

$$E_t\left(\frac{S_t}{S_{t+1}}\right) = \frac{(1+i_t^*)}{(1+i_t)}$$

Sum up … aggregated version

- Consumption good producers

$$Y_t^C = \lambda^c \left(\frac{P_t}{P_t^Y} \right)^{\theta} C_t$$

$$M_t^C = \left(1 - \lambda^c\right) \left(\frac{P_t}{P_t^M} \right)^{\theta} C_t$$

$$\left(\frac{p_t^*}{P_t} \right) = \left(\frac{\theta}{\theta - 1} \right) \frac{E_t \sum_{i=0}^{\infty} \omega^i \beta^i C_{t+i}^{1-\sigma} \varphi_{t+i} \left(\frac{P_{t+i}}{P_t} \right)^{\theta}}{E_t \sum_{i=0}^{\infty} \omega^i \beta^i C_{t+i}^{1-\sigma} \left(\frac{P_{t+i}}{P_t} \right)^{\theta-1}}$$

$$P_t^{1-\theta} = (1 - \omega)\left(p_t^*\right)^{1-\theta} + \omega P_{t-1}^{1-\theta}$$

Sum up … aggregated version

- Intermediate goods producers

$$Y_t = Z_t N_t$$

$$Y_t = Y_t^C + Y_t^X$$

$$\left(\frac{p_t^{y*}}{P_t^Y}\right) = \left(\frac{\theta}{\theta-1}\right)\frac{E_t\sum_{i=0}^{\infty}\omega_y^i\beta^i C_{t+i}^{1-\sigma}\varphi_{t+1}^y\left(\frac{P_{t+i}^Y}{P_t^Y}\right)^{\theta}}{E_t\sum_{i=0}^{\infty}\omega_y^i\beta^i C_{t+i}^{1-\sigma}\left(\frac{P_{t+i}^Y}{P_t^Y}\right)^{\theta-1}}$$

$$\left(P_t^Y\right)^{1-\theta} = \left(1-\omega_y\right)\left(p_t^{Y*}\right)^{1-\theta} + \omega_y\left(P_{t-1}^Y\right)^{1-\theta}$$

Sum up … aggregated version

- Importers

$$M_t = M_t^C + M_t^Y$$

$$\left(\frac{p_t^{m*}}{P_t^M}\right) = \left(\frac{\theta}{\theta-1}\right)\frac{E_t\sum_{i=0}^{\infty}\omega_m^i\beta^i C_{t+i}^{1-\sigma}\varphi_{t+1}^m\left(\frac{P_{t+i}^M}{P_t^M}\right)^{\theta}}{E_t\sum_{i=0}^{\infty}\omega_m^i\beta^i C_{t+i}^{1-\sigma}\left(\frac{P_{t+i}^M}{P_t^M}\right)^{\theta-1}}$$

$$\left(P_t^M\right)^{1-\theta} = \left(1-\omega_y\right)\left(p_t^{m*}\right)^{1-\theta} + \omega_y\left(P_{t-1}^M\right)^{1-\theta}$$

Sum up … aggregated version

- Exporters

$$X_t = \left(\frac{P_t^X S_t}{P_t^*}\right)^{\vartheta} M_t^*$$

$$\left(\frac{p_t^{x*}}{P_t^X}\right) = \left(\frac{\theta}{\theta-1}\right)\frac{E_t\sum_{i=0}^{\infty}\omega_x^i\beta^i C_{t+i}^{1-\sigma}\varphi_{t+1}^x\left(\frac{P_{t+i}^X}{P_t^X}\right)^{\theta}}{E_t\sum_{i=0}^{\infty}\omega_x^i\beta^i C_{t+i}^{1-\sigma}\left(\frac{P_{t+i}^X}{P_t^X}\right)^{\theta-1}}$$

$$\left(P_t^X\right)^{1-\theta} = \left(1-\omega_x\right)\left(p_t^{x*}\right)^{1-\theta} + \omega_x\left(P_{t-1}^X\right)^{1-\theta}$$

Sum up … aggregated version

- Current account, net external position, risk premium and UIP

$$B_t = B_{t-1}\left(1+i_t^*\right)\left(1+\Delta S_t\right)+P_t^X X_t - P_t^M M_t$$

$$b_t = b_{t-1}\left(1+i_t^*\right)\frac{P_{t-1}^X X_{t-1}}{P_t^X X_t}+1-\frac{P_t^M M_t}{P_t^X X_t}$$

$$prem_t = -\xi b_t$$

$$s_{t+1} - s_t = i_t - i_t^* - prem_t$$

Sum up …

- Equations above represent a system of non-linear forward-looking equations
- It is impossible to handle them in that form
- To find a solution, i.e. a system of equations where variables do not depend on future values, equations above have to be approximated
- The most common approximation is a first order Taylor approximation, i.e. a linear approximation around a fixed point
- We know how to solve a system of linear forward-looking difference equations

Log-linear approximation of household's f.o.c.

- Approximation of the Euler equation around the zero steady state:

$$\hat{c}_t = E_t \hat{c}_{t+1} - \left(\frac{1}{\sigma}\right)\left(\hat{i}_t - E_t \pi_{t+1}\right)$$

- Approximation of the intratemporal (labor market) first order condition:

$$\eta \hat{n}_t + \sigma \hat{c}_t = \hat{w}_t - \hat{p}_t$$

- Real wage is equal to the MRS between leisure and consumption
- Labor market is always cleared in this version!

Log-linear approximation of household's f.o.c.

- Approximating the third first order condition:

$$\hat{m}_t - \hat{p}_t = \left(\frac{1}{bi^{ss}}\right)\left(\sigma\hat{c}_t - \hat{i}_t\right)$$

- We get a money demand equation!
  - For a given interest rate, consumption and price level one can calculate the implied money using the equation above.
  - But money does not appear in IS curve or in the Phillips curve!
  - This is because money and consumption are separable in the utility function
  - Money is needed for transactions but is not a driving force for the model

Log-linear approximation of firms' price setting (general)

- Using the pricing function of firms and the price aggregation

$$\left(\frac{p_t^*}{P_t}\right) = \left(\frac{\theta}{\theta-1}\right) \frac{E_t \sum_{i=0}^{\infty} \omega^i \beta^i C_{t+i}^{1-\sigma} \varphi_{t+i} \left(\frac{P_{t+i}}{P_t}\right)^{\theta}}{E_t \sum_{i=0}^{\infty} \omega^i \beta^i C_{t+i}^{1-\sigma} \left(\frac{P_{t+i}}{P_t}\right)^{\theta-1}}$$

$$P_t^{1-\theta} = (1-\omega)\left(p_t^*\right)^{1-\theta} + \omega P_{t-1}^{1-\theta}$$

- And quite terrible math …
- One gets the so called New Keynesian Phillips curve

New Keynesian Phillips curve

$$\pi_t = \beta E_t \pi_{t+1} + \tilde{\kappa}\hat{\varphi}_t \qquad \tilde{\kappa} = \frac{(1-\omega)(1-\beta\omega)}{\omega}$$

- Forward looking inflation process
  - When a firm sets its price it must consider future inflation because it may not be able to re-set its price for a while
- Real marginal cost is an important variable driving the inflation process
- The weight on inflation expectations vs. real marginal cost depends on the degree of price stickiness $\uparrow \omega \Rightarrow \tilde{\kappa} \uparrow$

Policy rule

- To close the model we need an interest rate rule.
  - But not just ANY rule!
  - It needs to satisfy certain conditions to avoid unstable dynamics or multiple equilibrium
- Example of an unstable rule:

$$\hat{i}_t = \rho_r \hat{i}_{t-1} + \nu_t$$

Policy rule

- Will not get into technical details but there are multiple solutions when the above rule is used!
  - Sunspot equilibria are possible
    - To see this think about what would happen if inflation expectations were to rise
    - Since the rule does not have inflation on the RHS, the real interest rate will fall
    - A decline in real interest rate leads to an increase in output gap
    - An increase in output gap increases actual inflation
    - Self-fulfilling high inflation!
- In general exogenous policy rules have this problem. Better to use rules that depend on endogenous variables (output gap and inflation)

Policy rule

- Setting a rule that would raise the nominal interest rate enough such that real interest would increase would be enough to solve the multiple equilibria problem! For example:

$$\hat{i}_t = \delta\pi_t + v_t$$

- A unique equilibrium exists as long as $\delta > 1$
- This is called the "Taylor principle"
- Taylor was the first to emphasize that the nominal interest rates should increase more than one-to-one in response to inflation

Policy rule

- The most common rule: “Taylor rule”

$$\hat{i}_t = \delta_\pi \pi_t + \delta_x x_t + v_t$$

- Taylor rule was proposed by Taylor in an empirical context.
  - When looked at how the US Fed set the interest rates historically it looked like they were following a rule that looked very much like what Taylor proposed
- Has been shown to provide a reasonable empirical description of the many Central Banks’ behavior

Policy rule

- An extension is “Forward looking Taylor rule”

$$\hat{i}_t = \delta_\pi E_t \pi_{t+1} + \delta_x x_t + \nu_t$$

- Policymaker responds to expected inflation as opposed to contemporaneous inflation.
  - It nests the Taylor rule as a special case
- With this type of rule and given the rest of the model, the condition to ensure unique equilibrium is:

$$\kappa(\delta_\pi - 1) + (1-\beta)\delta_x > 0$$

Extension – Inflation persistence

- Christiano, Eichenbaum and Evans (2005)
- Extension of Calvo pricing
  - Each period, firms that adjust their price are randomly selected
    - With probability $1-\omega$ firm can adjust price
    - With probability ω it indexes based on past inflation

$$p_{jt} = \pi_{t-1} p_{jt-1}$$

- Aggregate price index becomes

$$P_t^{1-\theta} = (1-\omega)\left(p_t^*\right)^{1-\theta} + \omega \pi_{t-1} P_{t-1}^{1-\theta}$$

Extension – Inflation persistence

- And the Phillips curve looks as:

$$\pi_t = \left(\frac{1}{1+\beta}\right)\pi_{t-1} + \left(\frac{\beta}{1+\beta}\right)E_t\pi_{t+1} + \frac{(1-\beta\omega)(1+\omega)}{(1+\beta)\omega}\hat{\varphi}_t$$

- Or

$$\pi_t = \left(\frac{1}{1+\beta}\right)\pi_{t-1} + \left(\frac{\beta}{1+\beta}\right)E_t\pi_{t+1} + (\eta+\sigma)\left[\frac{(1-\beta\omega)(1+\omega)}{(1+\beta)\omega}\right]x_t$$

- When approximating the real marginal cost by the output gap

Extension - Sticky wages

- Erceg, Henderson and Levin (1999) adopted the Calvo specification to sticky wages
- In the labor market households provide differentiated labor services and firms combine these different labor services to produce output
- Labor in the production function is a composite of individual types of labor services

$$N_t = \left[\int_0^1 n_{jt}^{\frac{\xi - 1}{\varsigma}} \, dj \right]^{\frac{\varsigma}{\varsigma - 1}}, \gamma > 1$$

Extension - Sticky wages

- Households face a labor demand that depends on wage they set relative to aggregate wage rate

$$n_{jt} = \left(\frac{W_{jt}}{W_t}\right)^{-\varsigma} N_t \qquad W_t = \left[\int_0^1 W_{jt}^{1-\varsigma} dj\right]^{\frac{1}{\varsigma-1}}$$

- Adding the CEE trick with indexation
  - With probability $1-\omega^W$ household can adjust wage
  - With probability ω^W it indexes based on past wage growth

$$W_{jt} = \pi_{t-1}^W W_{jt-1}$$

Extension - Sticky wages

- One can derive a "Wage" Phillips curve

$$\pi_t^W = \left(\frac{1}{1+\beta}\right)\pi_{t-1}^W + \left(\frac{\beta}{1+\beta}\right)E_t\pi_{t+1}^W + \frac{\left(1-\beta\omega^W\right)\left(1+\omega^W\right)}{\left(1+\beta\right)\omega^W}\hat{\varphi}_t^W$$

- Where $\hat{\varphi}_t^W$ is difference between the real wage and the MRS between the leisure and consumption
- Remember the households' intratemporal condition $\eta\hat{n}_t + \sigma\hat{c}_t = \hat{w}_t - \hat{p}_t$
- With sticky wages this one is not met at every period and a non zero $\hat{\varphi}_t^W$ emerges

Extension - Sticky wages

- Introduction of a “Wage” Phillips curve means that the real marginal costs in a “Price” Phillips curve cannot be approximated by the output gap any more
- One has to use

$$\pi_t = \left(\frac{1}{1+\beta}\right)\pi_{t-1} + \left(\frac{\beta}{1+\beta}\right)E_t\pi_{t+1} + \frac{(1-\beta\omega)(1+\omega)}{(1+\beta)\omega}\hat{\varphi}_t$$

- Where $\varphi_t = \frac{W_t / P_t}{Z_t}$ and $\hat{\varphi}_t = \hat{w}_t - \hat{p}_t - \hat{z}_t$

Extension – Households' habit formation

- Households utility function in respect to consumption can be specified as:

$$\frac{\left(C_{t+i} / H_{t+i}^{\gamma}\right)^{1-\sigma}}{1-\sigma}$$

- Where $H_t = C_{t-1}$ stays for the households habit formation
- A myopic behavior which is sometimes called a "keep up with Jones's" behavior
- Households keep their consumption related to certain reference value, usually a previous value

Extension – Households' habit formation

- Households' first order condition change for

$$\left(C_t/C_{t-1}^{\gamma}\right)^{-\sigma} = \beta(1+i_t)E_t\left(\frac{P_t}{P_{t+1}}\right)\left(C_{t+1}/C_t^{\gamma}\right)^{-\sigma}$$

$$\frac{\chi N_t^{\eta}}{\left(C_t/C_{t-1}^{\gamma}\right)^{-\sigma}} = \left(\frac{W_t}{P_t}\right)$$

- Log-linearized

$$\hat{c}_t = \left(\frac{1}{1+\gamma}\right)E_t\hat{c}_{t+1} + \left(\frac{\gamma}{1+\gamma}\right)\hat{c}_{t-1} - \frac{1}{\sigma(1+\gamma)}\left(\hat{i}_t - E_t\pi_{t+1}\right)$$

$$\eta\hat{n}_t + \sigma\hat{c}_t - \sigma\gamma\hat{c}_{t-1} = \hat{w}_t - \hat{p}_t$$

Issues/comments

- Today it is popular to say that the economic paradigm on which the DGE models are based failed to predict the recent crises and is therefore of no use for policy analysis
- That is wrong in principle
  - If the crises was predictable it would not happen
- Models help to design a policy reaction to shocks we are able to identify and understand in the real time
- They cannot help us with shocks we do not see and/or understand
- No model (and monetary economist) told banks to provide credits to people who evidently could not pay them back

Krugman view and similar …

- *… "Few economists saw our current crisis coming" and "…economists, as a group, mistook beauty, clad in impressive-looking mathematics, for truth"*

- It is an overstatement
- Indeed, many economists inside the IMF and central banks talked about housing and credit bubbles, etc.
- But it is very difficult to stop the party when majority of agents think bottles are bottomless

Issues/comments

- The framework was further extended in many ways
- It is common to incorporate investment and capital accumulation as well as government spending
- There are models with
  - Financial intermediaries and financial market imperfections
  - Housing sector
  - Heterogenous agents
- The question is whether it is worth the effort

Issues/comments

- All articles published on extensions show that the model performance and/or policy reaction do not change much under standard business cycle fluctuations
- While in a bust phase of the bubble a model with any kind of financial friction shows stronger drop in investment activity, real GDP and a need for more profound decline in interest rate
- That can be achieved with a persistent negative shock in investment
- And without introduction of complicated block of financial sector

Models used at Central Banks

- Many Central Banks have some kind of a DSGE model (or several of them) already developed and running
  - Bank of Canada - TOTEM
  - BoE - BEQM
  - Czech National Bank (CNB) – g3
  - Norges Bank - NEMO
  - Riksbank - RAMSES
- From outside it is difficult to guess to what extent those determine the published forecast and guide the policy