# L-1. Intertemporal Trade in a Two-Period Model

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# What You Should Already Know…

# Current account deficit...

… is a result of exports falling short of imports.

$$CA_t = NX_t + r_t B_t = X_t - M_t + r_t B_t$$

… is a result of  savings falling short of investments.

$$Y_t + r_t B_t = C_t + G_t + I_t + NX_t + r_t B_t$$

$$\Rightarrow \; CA_t = S_t - I_t$$

# Neoclassical Synthesis

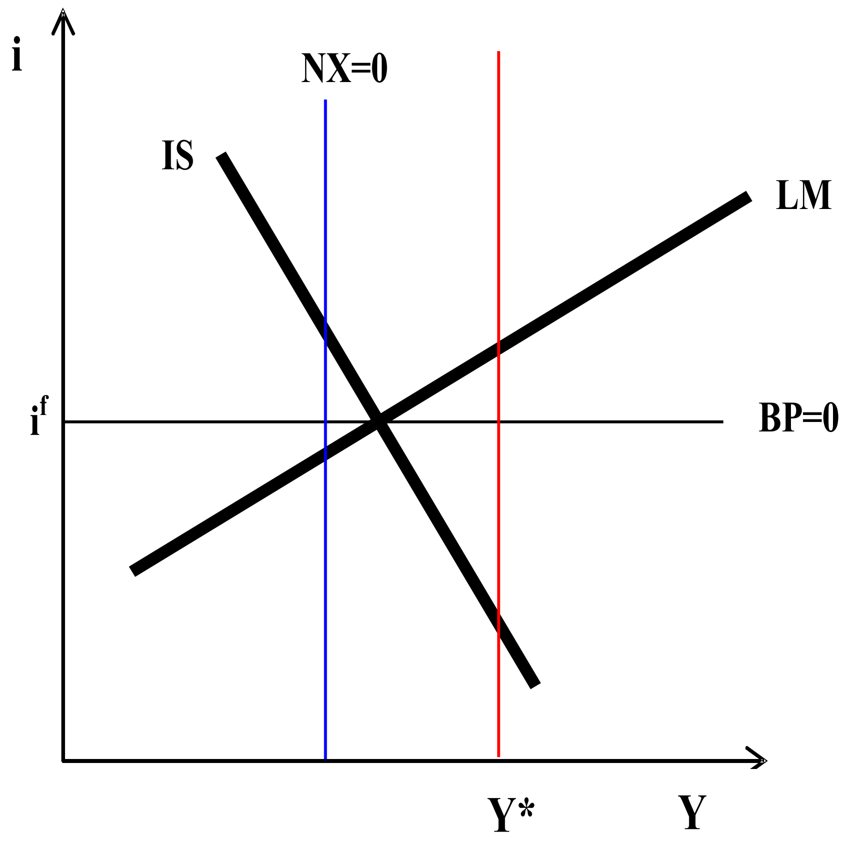
In these models, CA deficits reflects macroeconomic disequilibrium (economic overheating, low foreign demand, loss of price competitiveness).

$$NX_t = X\left(y_t^*; q_t\right) - M(y_t; q_t) = NX\left(\underset{-}{y_t}; \underset{+}{y_t^*}; \underset{+}{q_t}\right)$$

$$NX_t = \alpha_t - \beta y_t + \gamma y_t^* + \delta q_t$$

Safe level: CA deficit below 5 % of GDP.

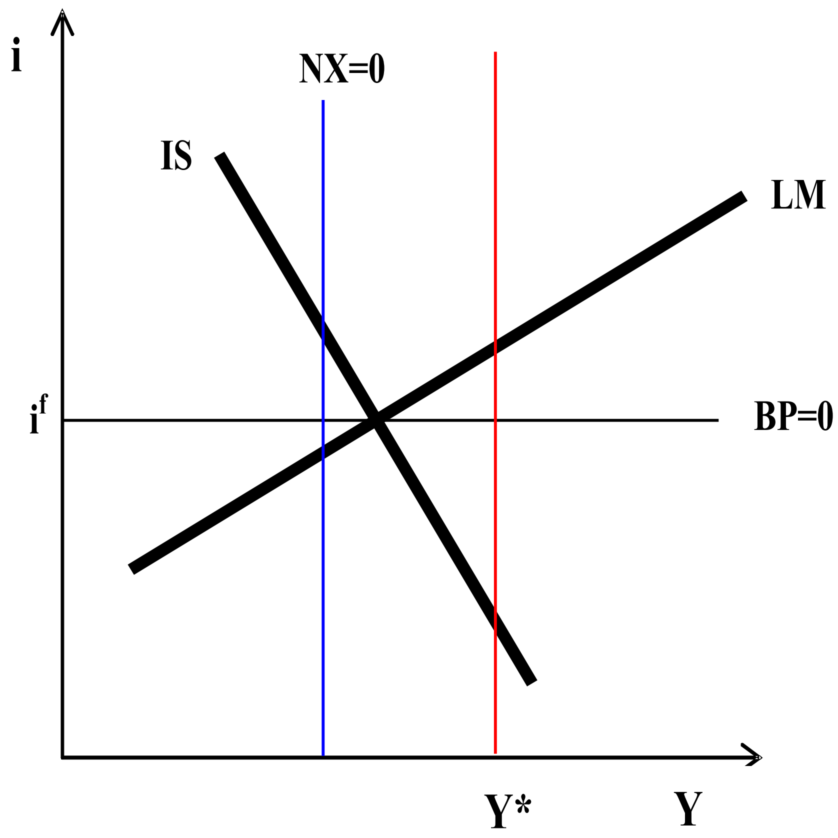
# Mundell-Fleming Model




Assume <u>perfect capital mobility</u> and <u>fixed ER</u>;

How to simultaneously achieve internal and external equilibrium?

# Mundell-Fleming Model




Assume perfect capital mobility and floating ER;

How to simultaneously achieve internal and external equilibrium?

# Mundell-Fleming Model: Shortcomings

Fixed prices (fixed money stock)

No explicit microfoundations;

Do discussion of the equilibrium and welfare issues;

No explicit treatment of expectations;

Comparative-static approach – no full-fledged dynamics.

e.g. Dornbusch – added dynamics and expectations ⇒ famous „ER overshooting hypothesis“ (see Lecture 6)

# Intertemporal Trade in a Two-Period Model

# Assumptions

Economy exists for two periods only = two periods lived households;
Single tradable good (real ER always equal to 1);
Prices fully flexible;
Income of a representative household $Y_1$, $Y_2$ falls down from heaven;
The good is non-storable (no investment);
No assets or debt at the beginning of period 1.

# Households' optimization problem

$$Max U_1^i = u\left(c_1^i\right) + \beta u\left(c_2^i\right)$$

$$s.t \quad c_1^i + \frac{c_2^i}{1+r} = y_1^i + \frac{y_2^i}{1+r}$$

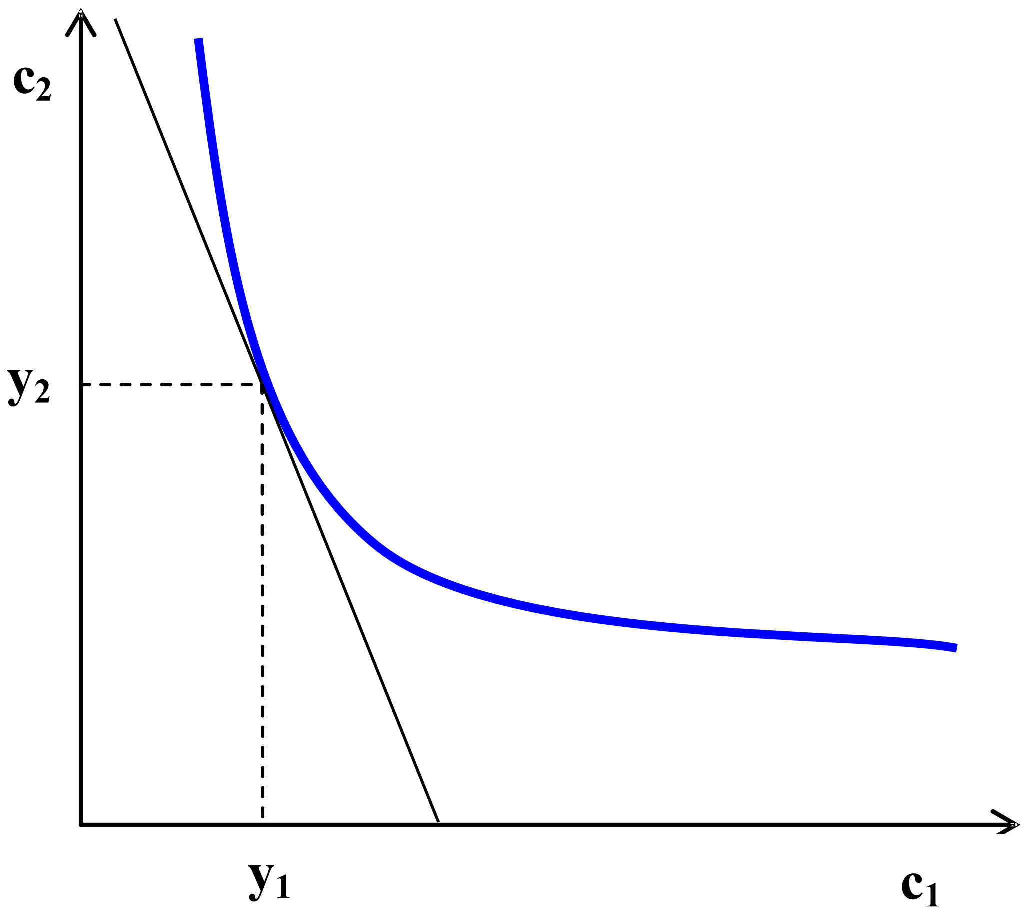
$$L = u\left(c_1^i\right) + \beta u\left(c_2^i\right) + \lambda\left[y_1^i + \frac{y_2^i}{1+r} - c_1^i - \frac{c_2^i}{1+r}\right]$$

$$\frac{\delta L}{\delta c_1} : u'\left(c_1^i\right) - \lambda = 0 \Rightarrow u'\left(c_1^i\right) = \lambda$$

$$\frac{\delta L}{\delta c_2} : \beta u'\left(c_2^i\right) - \frac{\lambda}{1+r} = 0 \Rightarrow \beta u'\left(c_2^i\right) = \frac{u'\left(c_1^i\right)}{1+r}$$

$$u'\left(c_1^i\right) = u'\left(c_2^i\right)\beta\left(1+r\right)$$

# Closed Economy




In equilibrium,
a closed economy must consume its income;

If $Y_1 << Y_2$, everyone wants to borrow $\Rightarrow$ interest rate jumps up

$$u'\left(c_1^i\right) = \left(1 + r\right)\beta\, u'\left(c_2^i\right)$$

$$c_1 = y_1;\ c_2 = y_2$$

# Closed Economy - example

$$y_1^i = 10; \quad y_2^i = 100; \quad \beta = \frac{1}{1.05}$$

$$U_1^i = \ln\left(c_1^i\right) + \beta \ln\left(c_2^i\right)$$

$$\frac{c_2^i}{c_1^i} = \left(1 + r\right)\beta$$

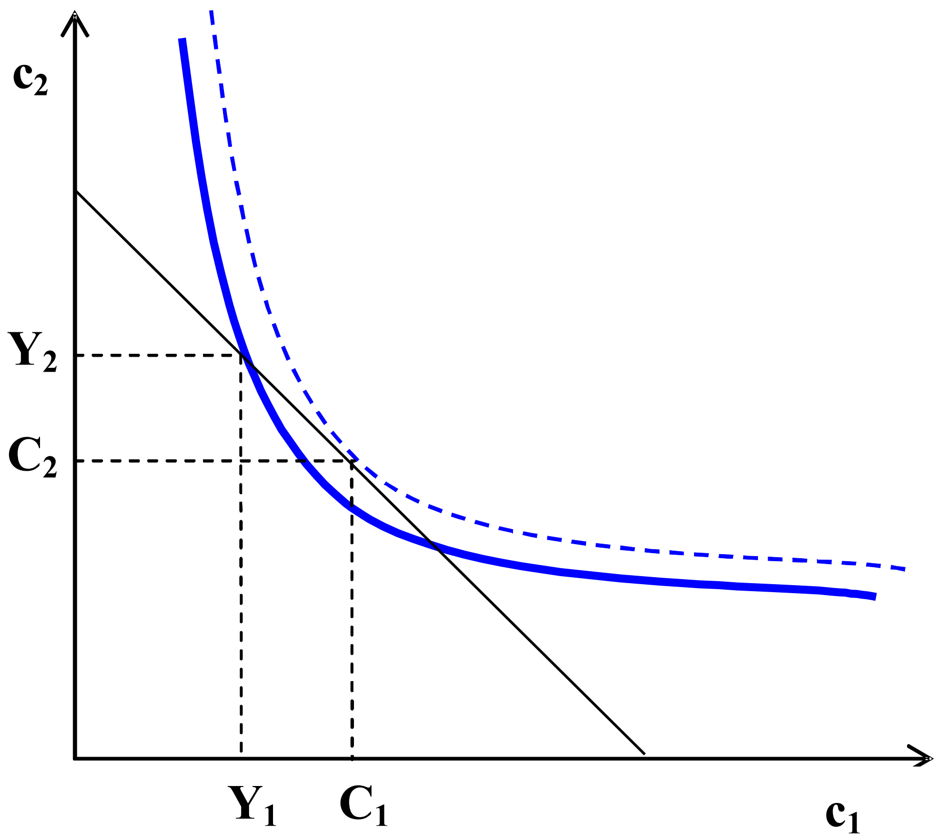
$$c_1 = y_1; \; c_2 = y_2$$

$$\frac{100}{10} = \left(1 + r\right)\frac{1}{1.05} \Rightarrow \quad \left(1 + r\right) = 10.5!!!$$

$$U_1^i = \ln\left(10\right) + \frac{1}{1.05}\ln\left(100\right) \cong 6.69$$

The desire to smooth consumption is very strong, the uneven pattern of income drives interest rates incredibly high.

# Small Open Economy




Can borrow at world interest
rate r*;

This helps smooth the consumption and increase welfare;

Debt must be repaid by surplus in period 2.

$$u'(c_1) = (1 + r^*)\beta u'(c_2)$$

$$Y_1 - C_1 = NX_1 = CA_1;\ Y_2 - C_2 = NX_2;\ CA_2 = NX_2 + r^* CA_1 = -CA_1;$$

# Open Economy - example

$$y_1^i = 10; \quad y_2^i = 100; \quad \beta = \frac{1}{1.05}; \; r^* = 0.05$$

$$U_1^i = \ln\left(c_1^i\right) + \beta \ln\left(c_2^i\right) \quad \left(\theta \to 1\right)$$

$$\frac{c_2^i}{c_1^i} = \left(1 + r^*\right)\beta = 1$$

$$c_1^i = \frac{1}{1+\beta}\left[y_1^i + \frac{y_2^i}{\left(1 + r^*\right)}\right] = \frac{1.05}{2.05}\left[10 + \frac{100}{1.05}\right] \cong 53.9$$

$$U_1^i \cong \ln\left(53.9\right) + \frac{1}{1.05}\ln\left(53.9\right) \cong 7.78$$

$$NX_1 = CA_1 \cong 10 - 53.9 \cong -43.9!!!$$

$$NX_2 \cong 100 - 53.9 \cong 46.1; \; CA_2 \cong 46{,}1 - 0.05 * 43.9 \cong 43.9$$

# Open Economy – extended solution

Utility function with constant elasticity of substitution, but not necessarily equal to 1

$$u\left(c^i\right) = \ln(c^i)$$

Constant and equal to 1 elasticity of substitution

$$u\left(c^i\right) = \frac{c^{1-\theta}}{1-\theta}$$

Constant but eventually ≠1 elasticity of substitution

$1/\theta$ Elasticity of substitution

# Open Economy – extended solution

$$u\left(c^i\right) = \frac{\left(c^i\right)^{1-\theta} - 1}{1-\theta}$$

$$\frac{c_2^i}{c_1^i} = \left[\left(1+r^*\right)\beta\right]^{\frac{1}{\theta}}$$

$$c_2^i = \left[\left(1+r^*\right)\beta\right]^{\frac{1}{\theta}} c_1^i$$

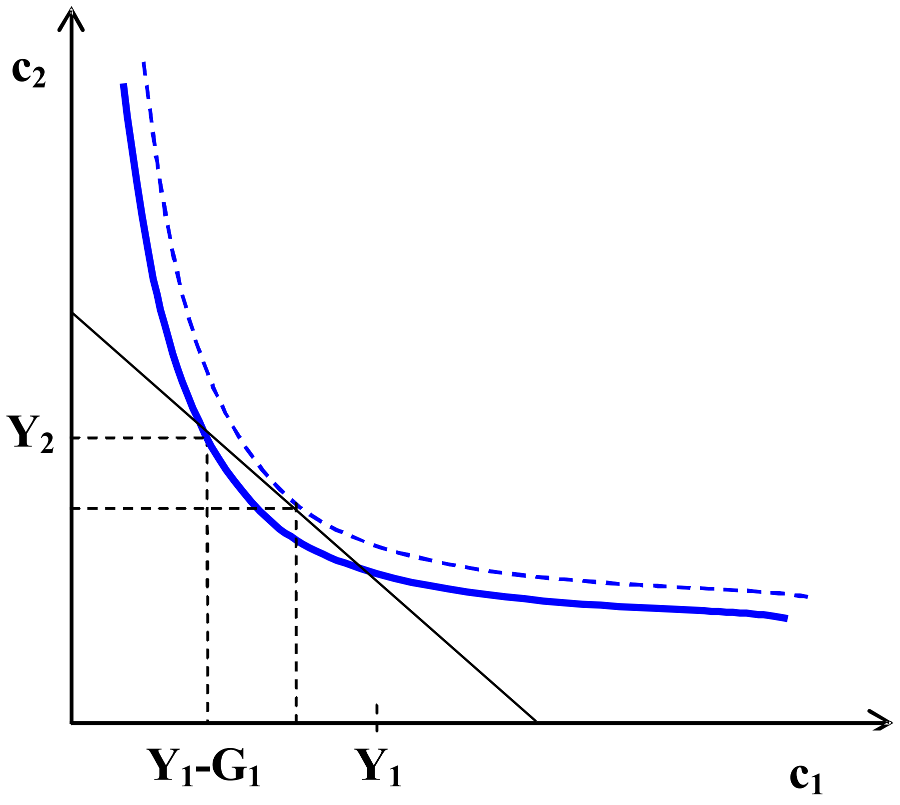
$$\left(y_1^i - c_1^i\right)\left(1+r^*\right) + y_2^i = \left[\left(1+r^*\right)\beta\right]^{\frac{1}{\theta}} c_1^i$$

$$c_1^i = \frac{1}{1+\left(1+r^*\right)^{\frac{1}{\theta}-1}\beta^{\frac{1}{\theta}}}\left[y_1^i + \frac{y_2^i}{\left(1+r^*\right)}\right]$$

Consumption depends on lifetime income;

Impact of r*:
(i) substitution effect $(1+r^*)^{1/\theta}$ ;
(ii) income effect $(1+r^*)^{-1}$;
(iii) wealth effect.

# Government Consumption




If the government consumption is high in period 1, country borrows abroad;

Government spending does not enter households utility

Ricardian equivalence holds.

$$MaxU(c_1;c_2) = u(c_1) + \beta u(c_2)$$

$$s.t \quad c_1 + \frac{c_2}{1+r^*} = (Y_1 - G_1) + \frac{Y_2 - G_2}{1+r^*}$$

# Productive Economy

$$\underset{C_1, I_1}{Max}\, U_1 = u\left(C_1\right) + \beta\, u\left(C_2\right)$$

$$Y = F\left(K\right);\; F'\left(K\right) > 0;\; F''\left(K\right) < 0;\; F\left(0\right) = 0$$

$$C_1 + I_1 + \frac{C_2 + I_2}{1 + r^*} = Y_1 - G_1 + \frac{Y_2 - G_2}{1 + r^*}$$

$$I_2 = -K_2$$

# Productive Economy

$$C_2 = (1 + r^*)[Y_1 - G_1 - C_1 - I_1] + Y_2 - G_2 + K_2$$

$$Y_2 = F(K_2) = F(K_1 + I_1)$$

$$MaxU(C_1; I_1) = u(c_1^i) + \beta u((1 + r^*)[Y_1 - G_1 - C_1 - I_1] + F(K_1 + I_1) - G_2 + (K_1 + I_1))$$

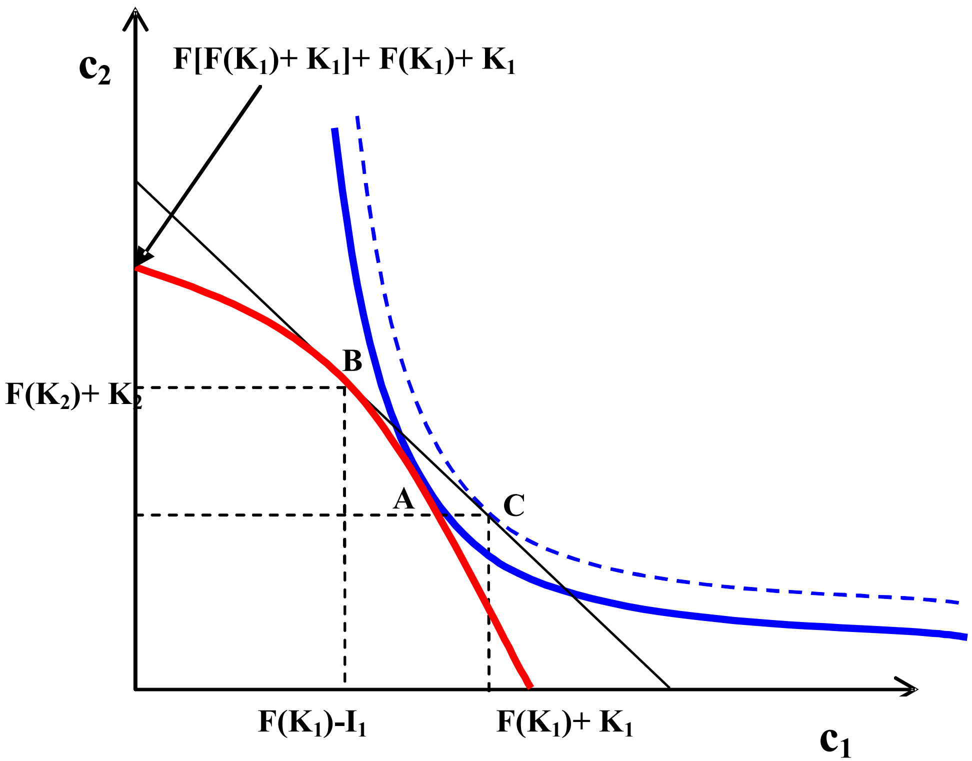
$$\frac{\delta U}{\delta C_1} : u'(c_1^i) - \beta u'(c_2^i)(1 + r^*) = 0$$

$$\frac{\delta U}{\delta I_1} : - \beta u'(c_2^i)(1 + r^*) + \beta u'(c_2^i) F'(K_2) + \beta u'(c_2^i) = 0$$

$$u'(c_1^i) = (1 + r^*)\beta u'(c_2^i)$$

$$F'(K_2) = r^*$$

# Productive Economy




Open economy can invest more than a closed one, but at the same time consume more in the 1st period.

$$u'\left(c_1^i\right) = \left(1 + r^*\right)\beta u'\left(c_2^i\right)$$

$$F'\left(K_2\right) = r^*$$

# Current Account Deficits

Contrary to the Mundell-Fleming model they are not evil !!!!!!
Help to smooth the consumption over time and thus increase welfare;
Help to isolate private consumption from jumps in government spending;
Allow the marginal product of capital to equalize world-wide, and thus improve capital‘s allocation and economic welfare;

However, as you can guess this is not the end ...