A Classical Monetary Model

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Lecture II

Basic Facts

- Perfect competition (homogenous good)
- Fully flexible prices in all markets
- Very limited role to money - serves as a unit of account
  - Later we introduce money to utility function
- Introduces notation and assumptions on preferences and technology

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Households

- A representative households seeks to maximize objective function

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) \tag{1}$$

  - Utility function has common properties
- Subject to a flow of budget constraints

$$P_t C_t + Q_t B_t = B_{t-1} + W_t N_t - T_t \tag{2}$$

  - B_t represents the quantity of one-period, nominally riskless discount bonds purchased in period t and maturing in period $t+1$
- Household is assumed to take as given the price of the good, the wage and the price of bonds
- And is subject to a solvency constraint

$$\lim_{t \to \infty} E_t\{B_t\} \geq 0 \tag{3}$$

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Optimal Consumption and Labor Supply

- Optimality conditions are given by

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} \quad (4)$$

$$Q_t = \beta E_t \left\{ \frac{U_{c,t+1}}{U_{c,t}} \frac{P_t}{P_{t+1}} \right\} \quad (5)$$

- Assuming that the period utility function takes the form

$$U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \quad (6)$$

- Optimality conditions become

$$C_t^{\sigma} N_t^{\varphi} = \frac{W_t}{P_t} \quad (7)$$

$$Q_t = \beta E_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right\} \quad (8)$$

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Firms

- A representative firm uses production function (technology)

$$Y_t = A_t N_t^{1-\alpha} \tag{9}$$

- Each period the firm maximizes profit

$$P_t Y_t - W_t N_t \tag{10}$$

  - Subject to production function and taking the price and wage as given
- Profit maximization results in optimality condition

$$\frac{W_t}{P_t} = (1-\alpha) A_t N_t^{-\alpha} \tag{11}$$

- Firm hires labor up to the point where its marginal product equals the real wage
- Equivalently, the marginal cost must be equated to the price P_t

$$P_t = \frac{W_t}{(1-\alpha) A_t N_t^{-\alpha}} \tag{12}$$

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Steady State and the Balanced Growth Path

- We assume constant growth of consumption and constant inflation in the steady state

$$\frac{\bar{C}_{t+1}^{-\sigma}}{\bar{C}_t^{-\sigma}} = \gamma \tag{13}$$

$$\frac{\bar{P}_{t+1}}{\bar{P}_t} = \bar{\Pi} \tag{14}$$

- Consumption growth (and the real wage) is driven by the exogenous growth in technology A

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- Steady-state level of consumption and the real wage are not constant and follow a balanced growth path

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Steady State Solution

- Households

$$\bar{C}^{\sigma}\bar{N}^{\varphi} = \frac{\bar{W}}{\bar{P}} \tag{16}$$

$$\bar{Q} = \beta\left\{\left(\frac{\bar{C}\gamma}{\bar{C}}\right)^{-\sigma}\frac{\bar{P}}{\bar{P}\bar{\Pi}}\right\} \tag{17}$$

$$\bar{Q} = \beta\gamma^{-\sigma}\frac{1}{\bar{\Pi}} \tag{18}$$

- Firms

$$\bar{Y} = \bar{A}\bar{N}^{1-\alpha} \tag{19}$$

$$\frac{\bar{W}}{\bar{P}} = (1-\alpha)\bar{A}\bar{N}^{-\alpha} \tag{20}$$

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Steady State Solution

- Steady state level of nominal and real interest rates is given by

$$\bar{Q} = \beta\gamma^{-\sigma}\frac{1}{\bar{\Pi}} \quad (21)$$

$$\bar{R} = \beta\gamma^{-\sigma} \quad (22)$$

  - And after taking *log*

$$\bar{i} = \rho + \sigma\gamma - \bar{\pi} \quad (23)$$

$$\bar{r} = \rho + \sigma\gamma \quad (24)$$

Steady State Solution

- Steady state level of labor and production comes out combining (16), (19) and (20) and using $C = Y$

$$\bar{N} = \upsilon_{na}\bar{A}^{\psi_{na}} \quad (25)$$

$$\bar{Y} = \upsilon_{ya}\bar{A}^{\psi_{ya}} \quad (26)$$

  - Where

$$\psi_{na} = \frac{1-\sigma}{\sigma(1-\alpha)+\phi+\alpha} \quad (27)$$

$$\psi_{ya} = \frac{1+\phi}{\sigma(1-\alpha)+\phi+\alpha} \quad (28)$$

$$\upsilon_{na} = (1-\alpha)^{\frac{1}{\sigma(1-\alpha)+\phi+\alpha}} \quad (29)$$

$$\upsilon_{ya} = \upsilon_{na}^{1-\alpha} \quad (30)$$

Log-linear approximation around the steady-state

- Definitions according to Uhlig (1999)

$$\tilde{x}_t = \ln X_t - \ln \bar{X} \quad (31)$$

$$X_t = \bar{X} e^{\tilde{x}_t} \quad (32)$$

- His rules are

$$e^{\tilde{x}_t} + a\tilde{y}_t \approx 1 + \tilde{x}_t + a\tilde{y}_t \quad (33)$$

$$\tilde{x}_t \tilde{y}_t \approx 0 \quad (34)$$

$$E_t\left[a e^{\tilde{x}_{t+1}}\right] \approx a + aE_t\left[\tilde{x}_{t+1}\right] \quad (35)$$

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Log-linear approximation around the steady-state

- Euler equation

$$
\begin{aligned}
Q_t &= \beta E_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right\} \\
\bar{Q} e^{q_t} &= \beta E_t \left\{ \left(\frac{\bar{C}^{-\sigma} \gamma^{-\sigma} e^{-\sigma c_{t+1}}}{\bar{C}^{-\sigma} e^{-\sigma c_t}} \right) \frac{\bar{P} e^{p_t}}{\bar{P} \bar{\bar{\Pi}} e^{p_{t+1}}} \right\} \\
\bar{Q} &= \beta \gamma^{-\sigma} \frac{1}{\bar{\bar{\Pi}}} \\
e^{q_t} &= \left(\frac{e^{-\sigma E_t\{c_{t+1}\}}}{e^{-\sigma c_t}} \right) \frac{e^{p_t}}{e^{E_t\{p_{t+1}\}}} \\
e^{q_t} &= e^{-\sigma E_t\{c_{t+1}\} + \sigma c_t + p_t - E_t\{p_{t+1}\}} \\
1 + q_t &= 1 - \sigma E_t\{c_{t+1}\} + \sigma c_t + p_t - E_t\{p_{t+1}\} \\
-i_t &= -\sigma E_t\{c_{t+1}\} + \sigma c_t + p_t - E_t\{p_{t+1}\} \\
c_t &= E_t\{c_{t+1}\} - \frac{1}{\sigma}(i_t - (E_t\{p_{t+1}\} - p_t))
\end{aligned}
$$

Log-linear approximation of optimality conditions

- Households

$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}\left(i_t - E_t\{\pi_{t+1}\}\right) \quad (36)$$

$$w_t - p_t = \sigma c_t + \varphi n_t \quad (37)$$

- Firms

$$w_t - p_t = a_t - \alpha n_t + \ln(1-\alpha) \quad (38)$$

$$y_t = a_t + (1-\alpha) n_t \quad (39)$$

- And for a moment somehow arbitrarily

$$m_t - p_t = y_t - \eta i_t \quad (40)$$

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Equilibrium

- In principle the Equilibrium is similar to the steady-state solution
- We start assuming $c_t = y_t$
- Then using the above and combining optimality conditions with the log-linear aggregate production function

$$n_t = \psi_{na} a_t + \upsilon_{na} \quad (41)$$

$$y_t = \psi_{ya} a_t + \upsilon_{ya} \quad (42)$$

- Where

$$\Psi_{na} = \frac{1-\sigma}{\sigma(1-\alpha)+\phi+\alpha} \quad (43)$$

$$\Psi_{ya} = \frac{1+\phi}{\sigma(1-\alpha)+\phi+\alpha} \quad (44)$$

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Equilibrium

- Furthermore, given the equlibrium process for output, Euler equation can be used to determine the implied real interest rate, $r_t \equiv i_t - E_t i_{t+1}$, as

$$r_t = \rho + \sigma E_t\{\Delta y_{t+1}\} \quad (47)$$
$$= \rho + \sigma E_t\{\Delta a_{t+1}\} \quad (48)$$

- Finally, the equilibrium real wage $\omega_t \equiv w_t - p_t$ is given by

$$\omega_t = a_t - \alpha n_t + log(1-\alpha) \quad (49)$$

- Note that the equilibrium dynamics of employment, output, and the real interest rate are determined *independently of monetary policy*
- All real variables fluctuate in response to variations in technology

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Equilibrium

- Output and the real wage always rise in the face of the productivity increase
- The sign of employment is ambiguous depending on whether σ is larger or smaller than one
  - When $\sigma < 1$, the substitution effect on labor supply resulting from a higher real wage dominates the negative effect caused by a smaller marginal utility of consumption
  - The opposite is true whenever $\sigma > 1$
  - And employment remains unchanged with $\sigma = 1$, i.e. when the utility of consumption is logarithmic
- Response of the real interest rate critically depends on the time series properties of technology
  - If the current improvement in technology is transitory, $E_t\{a_{t+1}\} < a_t$, then the real rate will go down
  - If the technology is expected to keep improving, $E_t\{a_{t+1}\} > a_t$, the real rate will increase with the rise in a_t

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Monetary Policy and Price Level Determination

- Not surprisingly, equilibrium behavior of nominal variables (inflation, nominal interest rate, nominal wage) cannot be determined uniquely by real forces
- It requires specification of how monetary policy is conducted
- Before we move to that recall the Fisher equation

$$i_t = r_t - E_t\{\pi_{t+1}\} \quad (50)$$

that implies that the nominal rate adjusts one for one with expected inflation given the real interest rate that is determined solely by real factors

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An Exogenous Path for the Nominal Interest Rate

- Consider first the case of the nominal interest rate following an *exogenous stationary process* i and assume that i_t has a mean ρ consistent with a steady state with zero inflation and no growth
- Using Fisher equation, write

$$E_t\{\pi_{t+1}\} = i_t - r_t \tag{51}$$

  where r_t is determined independently of monetary policy
- While previous equation pins down expected inflation there is no other condition determining current inflation. It follows that any path for the price level satisfying

$$p_{t+1} = p_t + i_t - r_t + \xi_t \tag{52}$$

  where ξ_t is a shock, possibly unrelated to economic fundamentals, satisfying $E_t\{\xi_{t+1}\} = 0$ for all t.
- Such shocks are referred as *sunspot shocks* and the example above shows how an exogenous nominal interest rate leads to *price level indeterminacy*

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Simple Inflation-Based Interest Rate Rule

- Suppose now that the central bank adjusts the nominal interest rate according to the rule
$$i_t = \rho + \Phi_\pi \pi_t \tag{53}$$
where $\Phi_\pi > 0$
- Combining this rule with Fisher equation yields
$$\Phi_\pi = E_t\{\pi_{t+1}\} + \hat{r}_t \tag{54}$$
where $\hat{r}_t \equiv \rho - r_t$
- If $\Phi_\pi > 1$, the previous difference equation has only one stationary solution (solution that remains in the vicinity of the steady state)
- Solution can be obtained by solving (54) forward
$$\pi = \sum_{k=0}^{\infty} \Phi_\pi^{-(k+1)} E_t\{\hat{r}_{t+k}\} \tag{55}$$
This equation fully determines inflation as a function of the path of the real interest rate.

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- In contrary, if $\Phi_\pi < 1$, the stationary solutions to (54) take the form

$$\pi_{t+1} = \Phi_\pi \pi_t - \hat{r}_t + \xi_{t+1} \tag{56}$$

where $\{\xi_{t+1}\}$ is, again, an arbitrary sequence of shocks, satisfying $E_t\{\xi_{t+1}\} = 0$ for all t

- It follows that any process $\{\pi_t\}$ satisfying previous equation is consistent with equilibrium
- As in the case of exogenous nominal rate the price level (and, hence, inflation) are not determined uniquely
- The condition for a determinate price level, Φ_π, is a property known as the *Taylor principle*

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An Exogenous Path for the Money Supply

- Now suppose that the central bank sets an exogenous path for the money supply $\{m_t\}$
- Eliminating interest rate in Fisher equation using the money demand yields following difference equation for the price level

$$p_t = \left(\frac{\eta}{1+\eta}\right) E_t\{p_{t+1}\} + \left(\frac{1}{1+\eta}\right) m_t + u_t \tag{57}$$

where $u_t \equiv (1+\eta)^{-1}(\eta r_t - y_t)$ evolves independently of $\{m_t\}$

- If $\eta > 0$ the above difference equation can be solved forward as

$$p_t = \frac{1}{1+\eta} \sum_{k=0}^{\infty} \left(\frac{\eta}{1+\eta}\right)^k E_t\{m_{t+k}\} + u'_t \tag{58}$$

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An Exogenous Path for the Money Supply

- Previous forward solution can be rewritten in terms of the expected future growth rate of money

$$p_t = m_t \frac{1}{1+\eta} \sum_{k=1}^{\infty} \left(\frac{\eta}{1+\eta} \right)^k E_t\{\Delta m_{t+k}\} + u_t' \tag{59}$$

  Hence, an arbitrary exogenous path for the money supply always determines the price level uniquely

An Exogenous Path for the Money Supply

- Finally, given the price level, money demand can be used to solve for the nominal interest rate

$$i_t = \eta^{-1}[y_t - (m_t - p_t)] \quad (60)$$

$$i_t = \eta^{-1} \sum_{k=1}^{\infty} \left(\frac{\eta}{1+\eta} \right)^k E_t\{\Delta m_{t+k}\} + u'_t \quad (61)$$

where $u''_t \equiv \eta^{-1}(u'_t + y_t)$ is independent of monetary policy

- In a basic classical model the households' utility is a function of consumption and hours only - two variables that are invariant to the way monetary policy is conducted
- That means that there is no policy rule better than any other - monetary policy causing huge fluctuation in nominal variables is no less desirable that one stabilizing prices in the face of the same shock
- Clearly, this result is extreme and empirically unappealing and we will overcome it with versions of the classical model in which a motive to keep part of the households' wealth in money is introduced explicitly

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