L-2. Infinite Horizon Model

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Assumptions

Economy and people (families) exist forever

Single tradable good (real ER always equal 1)

Prices fully flexible

Small open economy with perfect capital mobility

$$Max U_t = \sum_{s=t}^{\infty} \beta^{s-t} u(C_s); \quad \beta = \frac{1}{1+r^*}$$

$$s.t \quad \sum_{s=t}^{\infty} \left(\frac{1}{1+r^*}\right)^{s-t} (C_s + I_s) = (1+r^*)B_t + \sum_{s=t}^{\infty} \left(\frac{1}{1+r^*}\right)^{s-t} (Y_s - G_s)$$

$$\lim_{T \to \infty} \left(\frac{1}{1+r^*}\right)^{T} B_{t+T+1} = 0$$

$$Y_s = A_s F(K_s)$$

Solution for any *s* and *s+1* periods

$$MaxU_t = \sum_{s=t}^{\infty} \beta^{s-t} u(C_s); \quad \beta = \frac{1}{1+r^*}$$

$$s.t \quad C_s + I_s + B_{s+1} = (1+r^*)B_s + Y_s - G_s$$

$$Y_s = A_s F(K_s); K_{s+1} = K_s + I_s$$

$$\lim_{s\to\infty}\left(\frac{1}{1+r^*}\right)^s B_{t+s+1} = 0$$

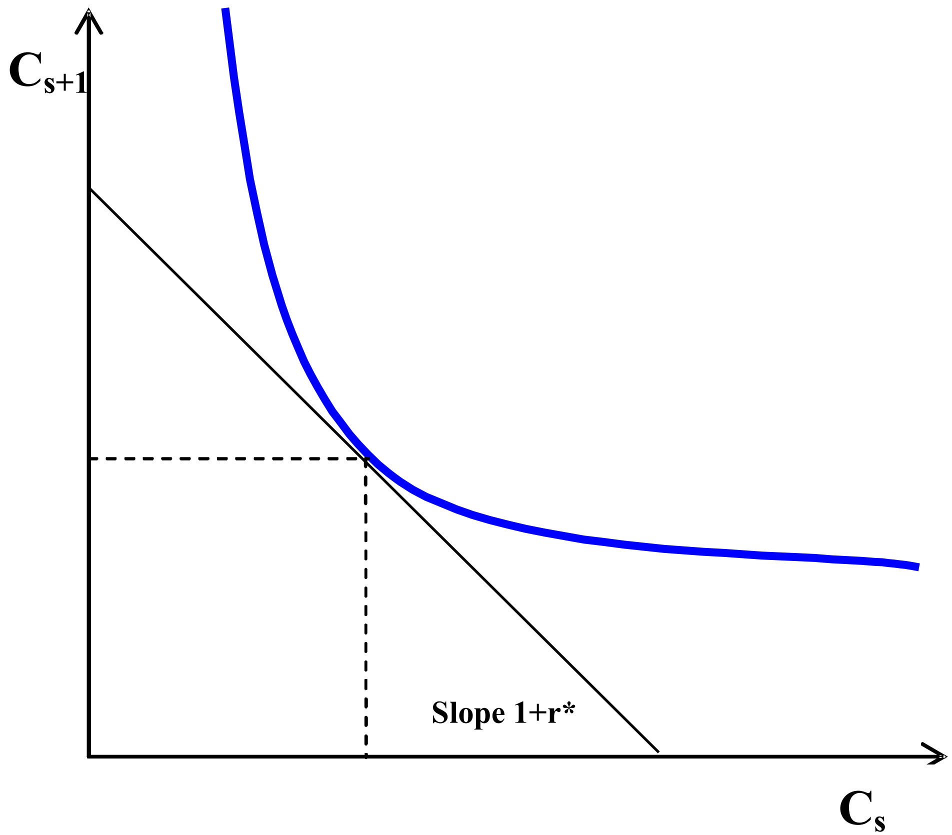
$$L_s = \sum_{s=t}^{\infty} \beta^{s-t}\left[u(C_s) + \lambda_s\left[(1+r^*)B_s + Y_s - G_s - C_s - I_s - B_{s+1}\right]\right]$$

$$\frac{\partial L_s}{\partial C_s} : u'(C_s) - \lambda_s = 0 \Rightarrow u'(C_s) = \lambda_s$$

$$\frac{\partial L_s}{\partial I_s} : \beta\lambda_{s+1}A_{s+1}F'(K_{s+1}) - \lambda_s = 0 \Rightarrow A_{s+1}F'(K_{s+1}) = \frac{\lambda_s}{\beta\lambda_{s+1}}$$

$$\frac{\partial L_s}{\partial B_{s+1}} : -\lambda_s + \beta\lambda_{s+1}(1+r^*) = 0 \Rightarrow u'(C_s) = \beta(1+r^*)u'(C_{s+1})$$

1st Order Conditions



Euler equation must hold for any two periods;

MPK=r* (we assume no depreciation of capital).

$$u'(c_s) = (1+r^*)\beta u'(c_{s+1})$$

$$A_{s+1}F'(K_{s+1}) = (1+r^*)$$

Consumption Level

$$\beta\left(1+r^*\right)=1\Rightarrow\quad\overline{C}$$

$$\sum_{s=t}^{\infty}\left(\frac{1}{1+r^*}\right)^{s-t}\left(\overline{C}+I_s\right)=\left(1+r^*\right)B_t+\sum_{s=t}^{\infty}\left(\frac{1}{1+r^*}\right)^{s-t}\left(Y_s-G_s\right)$$

$$\overline{C}=\frac{r^*}{1+r^*}\left[\left(1+r^*\right)B_t+\sum_{s=t}^{\infty}\left(\frac{1}{1+r^*}\right)^{s-t}\left(Y_s-G_s-I_s\right)\right]=\frac{r^*}{1+r^*}W_t$$

$$\sum_{s=t}^{\infty}q^{s-t}\overline{C}=\frac{1}{1-q}\overline{C}\qquad\left(q=\frac{1}{1+r^*};\ \frac{1}{1-q}=\frac{1}{1-\frac{1}{1+r^*}}=\frac{1+r^*}{r^*}\right)$$

Current Account

$$CA_t = Y_t + r^*B_t - C_t - G_t - I_t$$

$$CA_t = Y_t + r^*B_t - r^*B_t - \frac{r^*}{1+r^*}\left[\sum_{s=t}^{\infty}\left(\frac{1}{1+r^*}\right)^{s-t}\left(Y_s - G_s - I_s\right)\right] - G_t - I_t$$

$$CA_t = \left(Y_t - \tilde{Y}_t\right) - \left(G_t - \tilde{G}_t\right) - \left(I_t - \tilde{I}_t\right)$$

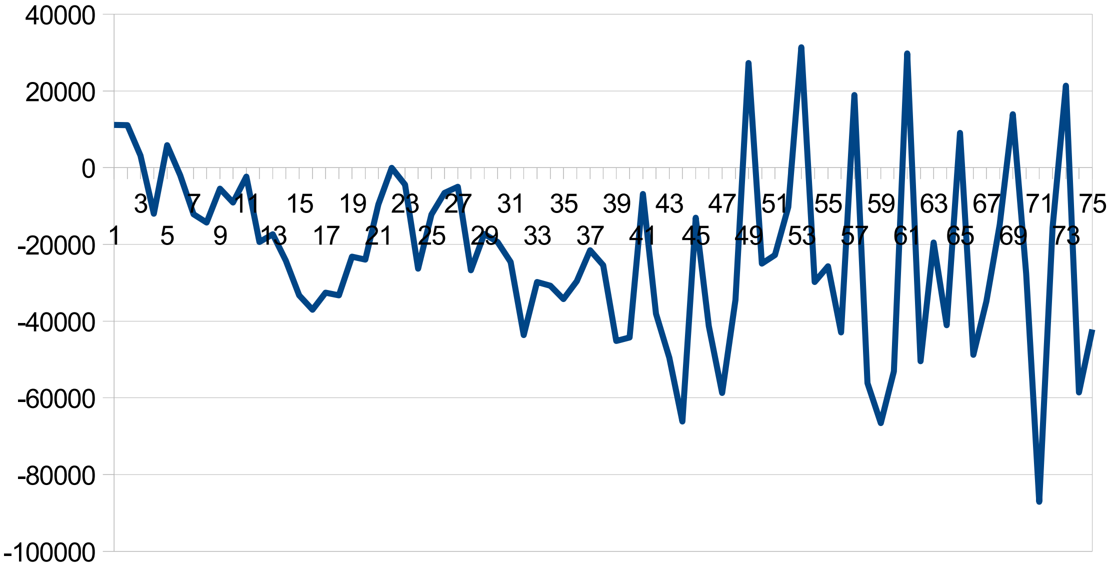
$$\tilde{X}_t \equiv \frac{r^*}{1+r^*}\sum_{s=t}^{\infty}\left(\frac{1}{1+r^*}\right)^{s-t} X_s \quad \text{...permanent level of a variable}$$

$$\sum_{s=t}^{\infty}\left(\frac{1}{1+r^*}\right)^{s-t}\tilde{X}_t \equiv \sum_{s=t}^{\infty}\left(\frac{1}{1+r^*}\right)^{s-t} X_s$$

When is the country bankrupt?

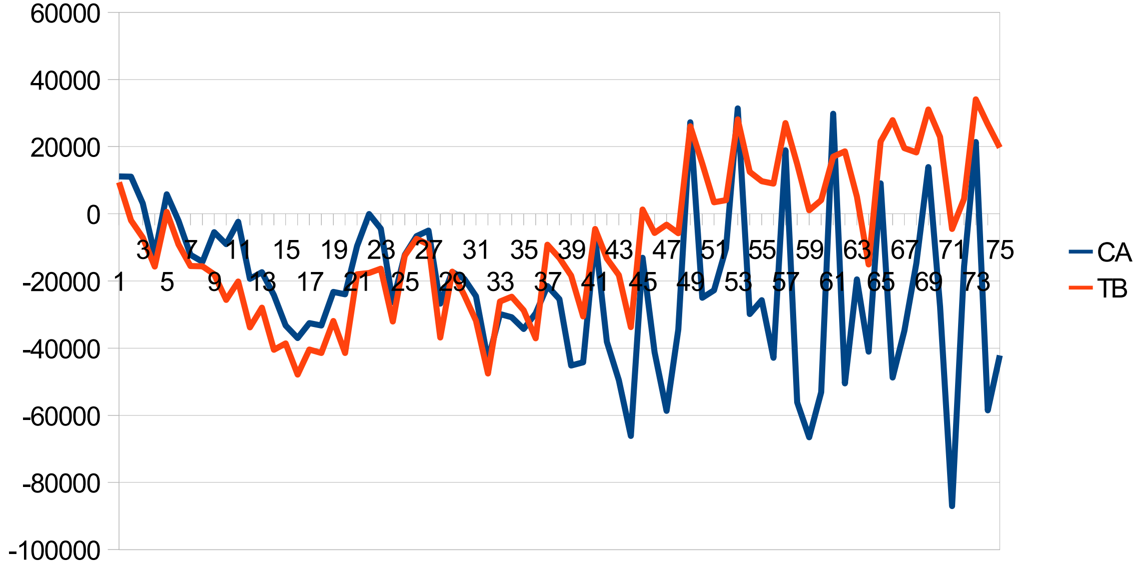
$$-\left(1+r^{*}\right)B_{t}=\sum_{s=t}^{\infty}\left(\frac{1}{1+r^{*}}\right)^{s-t}\left(Y_{s}-G_{s}-I_{s}-C_{s}\right)$$

When is the country bankrupt?



The Czech Republic has had a CA deficit since 1993

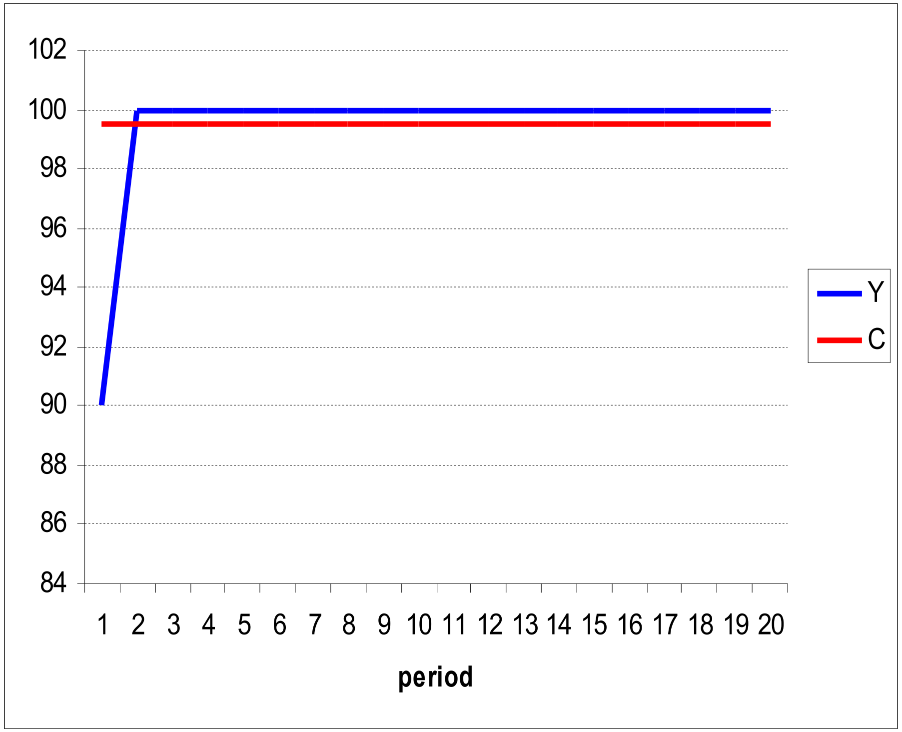
When is the country bankrupt?

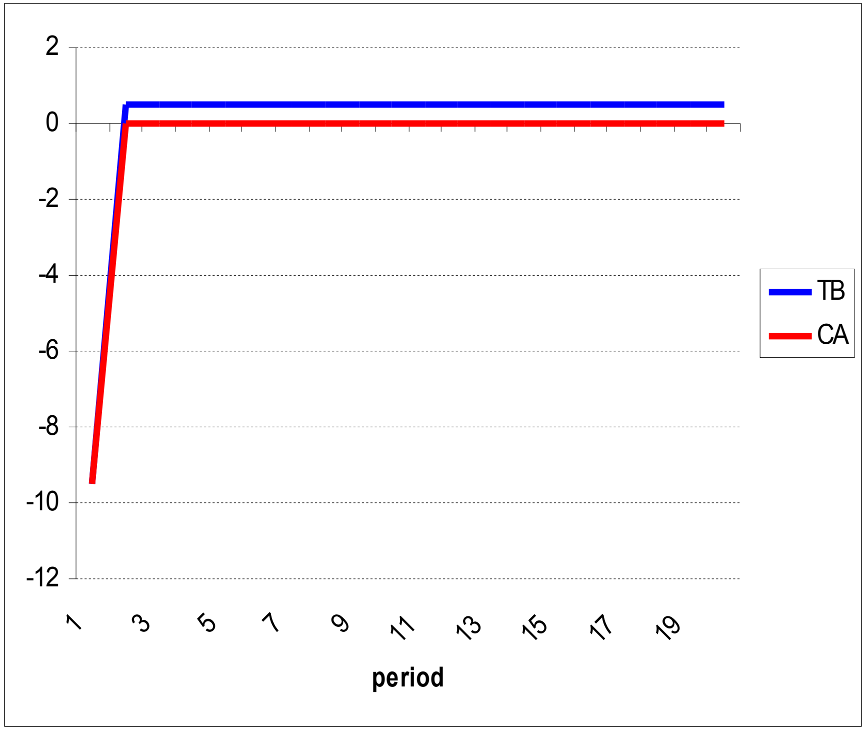


But accompanied by a positive trade balance since 2005

Example – One-off shock

$$Y_1 = 90;\ Y_{s\geq 2} = 100; I = G = 0;\ \beta = \frac{1}{1.05};\ r^* = 0.05;\ \sigma = 2;\ B_1 = 0$$



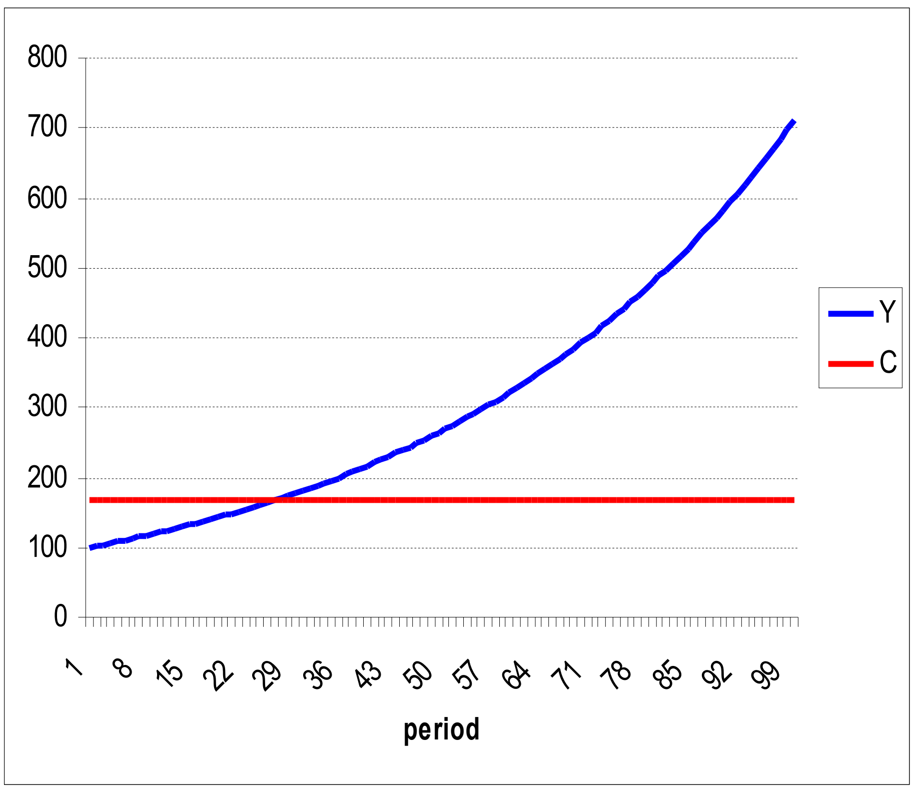


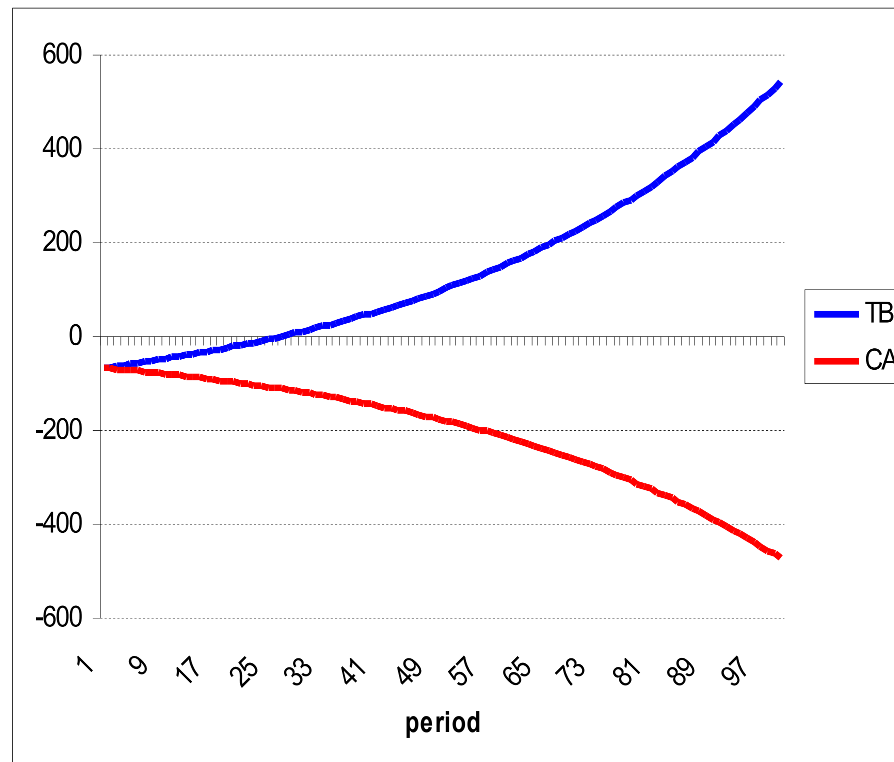
Current account deficits

Help to isolate private consumption from jumps in GDP, government spending and investment;
Allow the marginal product of capital to equalize world-wide;
However …

Example – Growing Economy

$$Y_1 = 100;\ I = G = 0;\ g = 2\%;\ \beta = \frac{1}{1.05};\ r^* = 0.05;\ \sigma = 2;\ B_1 = 0$$





Extension for (β(1+r)≠1)

$$\beta\left(1+r^*\right)\neq 1;\quad u(C)=\frac{C^{1-\theta}}{1-\theta}$$

$$C_{s+1}=\left[\left(1+r^*\right)\beta\right]^{\sigma}C_s;\quad \sigma\equiv 1/\theta$$

$$\sum_{s=t}^{\infty}\left(\frac{1}{1+r^*}\right)^{s-t}C_t\left[\left(1+r^*\right)\beta\right]^{\sigma(s-t)}=\left(1+r^*\right)B_t+\sum_{s=t}^{\infty}\left(\frac{1}{1+r^*}\right)^{s-t}\left(Y_s-G_s-I_s\right)$$

$$C_t=\frac{r^*+\left(1-\left[\left(1+r^*\right)\beta\right]^{\sigma}\right)}{1+r^*}\left[\left(1+r^*\right)B_t+\sum_{s=t}^{\infty}\left(\frac{1}{1+r^*}\right)^{s-t}\left(Y_s-G_s-I_s\right)\right]$$

$$\left(q=\frac{\left[\left(1+r^*\right)\beta\right]^{\sigma}}{1+r^*};\quad \frac{1}{1-q}=\frac{1}{1-\frac{\left[\left(1+r^*\right)\beta\right]^{\sigma}}{1+r^*}}=\frac{1+r^*}{r^*+1-\left[\left(1+r^*\right)\beta\right]^{\sigma}}\right)$$

Current Account

$$CA_t = Y_t + r^* B_t - C_t - G_t - I_t$$

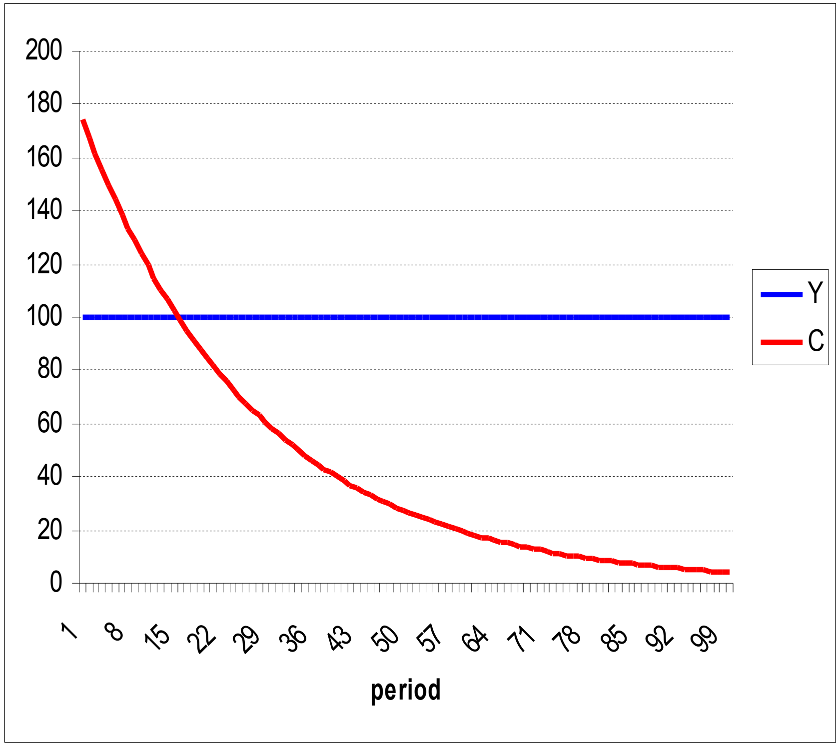
$$CA_t = Y_t + r^* B_t - \frac{r^* + \left(1 - \left[\left(1 + r^*\right)\beta\right]^{\sigma}\right)}{1 + r^*}\left[\left(1 + r^*\right)B_t + \sum_{s=t}^{\infty}\left(\frac{1}{1 + r^*}\right)^{s-t}\left(Y_s - G_s - I_s\right)\right] - G_t - I_t$$

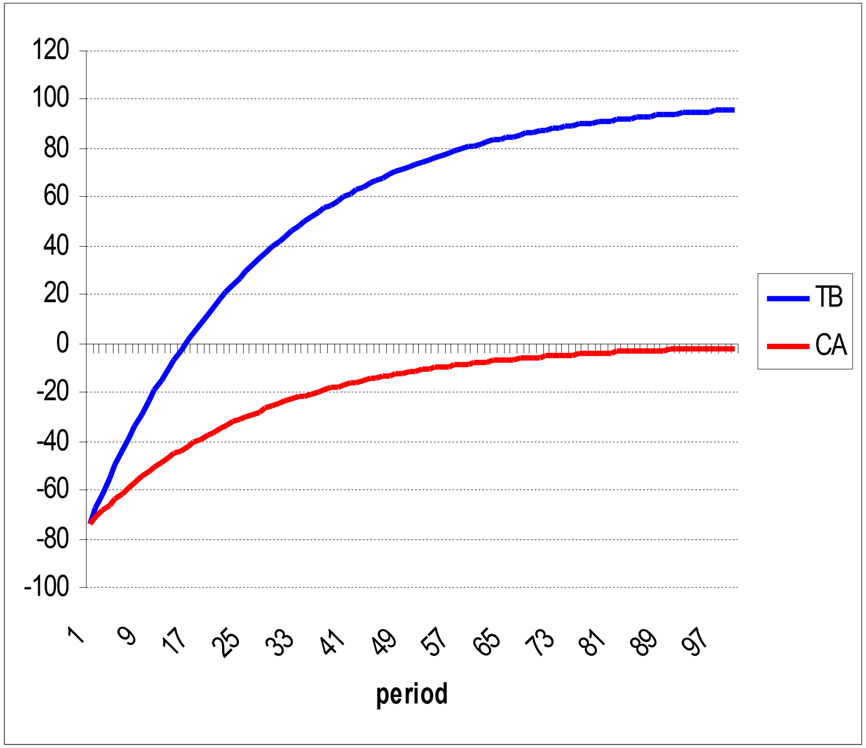
$$CA_t = \left(Y_t - \tilde{Y}_t\right) - \left(G_t - \tilde{G}_t\right) - \left(I_t - \tilde{I}_t\right) - \frac{1 - \left[\left(1 + r^*\right)\beta\right]^{\sigma}}{1 + r^*} W_t$$

If $\beta(1+r^*)<1$, initial current account is in a deficit even if all variables are at their permanent levels (C starts at a high level and then declines over time).

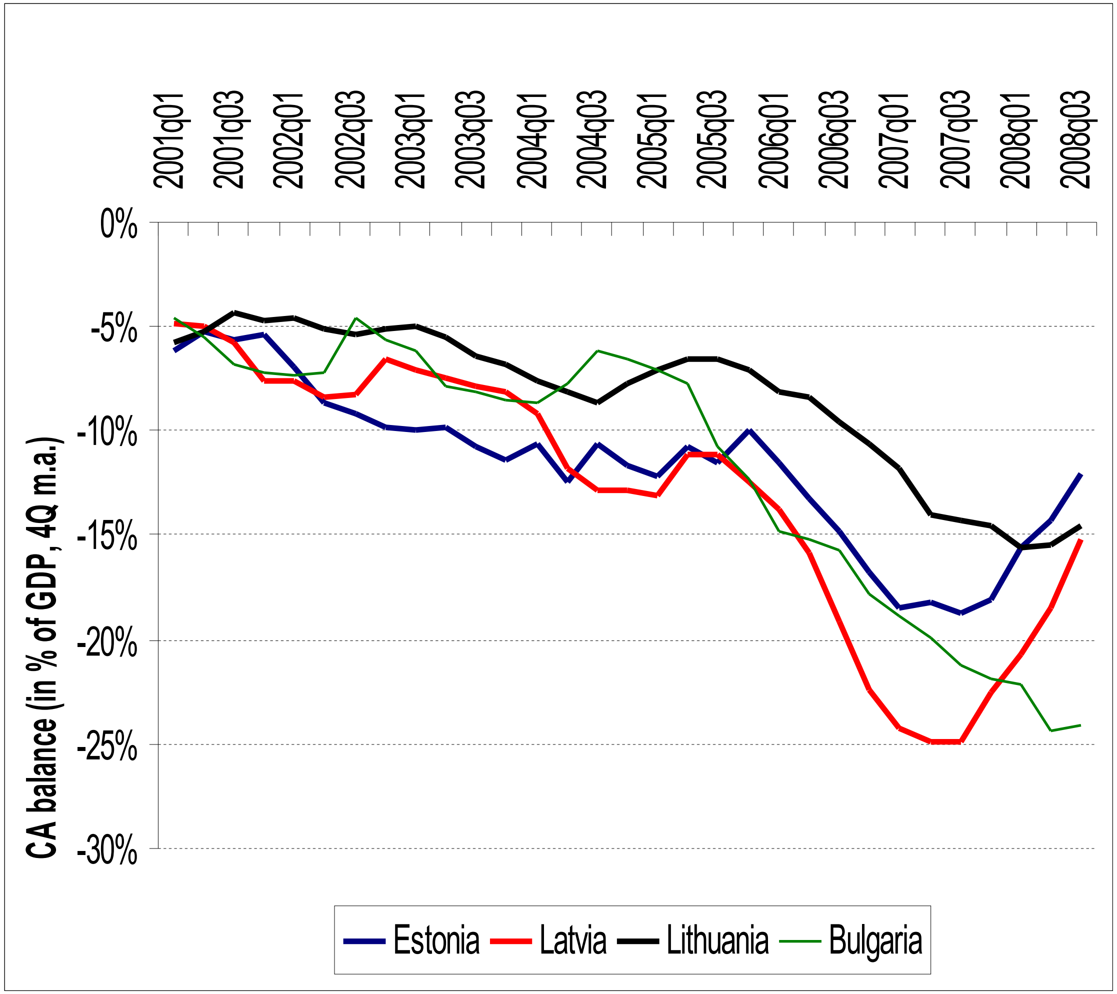
Example – Impatient People

$$Y = 100;\ I,G = 0;\ \beta = \frac{1}{1.07};\ r^* = 0.05;\ \sigma = 2;\ B_1 = 0$$





Is the prediction of large CA deficits realistic?



See what happened in the Baltics and Bulgaria.

For a while they were on an expansionary path

Summary

The model gives us some nice and intuitive predictions concerning the CA determination;
But we need some convenient assumptions to avoid problems;
Plausible deviations from these assumptions may lead to CA deficits/surpluses which might not be plausible in reality (in terms of their size; but what about the Baltic States in recent years?).