A Classical Monetary Model - Money in the Utility Function

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Lecture III

Basic Facts

- So far, the only role played by money was to serve as a numeŕaire, i.e. we dealt with a *cashless economy*
- We incorporate a role of money other than that of unit of account to generate demand for money
- We assume that real balances are an argument in the utility function
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  - Be aware that there are other options such as cash-in-advance

Households

- Households' preferences are now given by

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, \frac{M_t}{P_t}, N_t) \tag{1}$$

  - Period utility is increasing and concave in real balances $\frac{M_t}{P_t}$
- And the flow budget constraint takes the form

$$P_t C_t + Q_t B_t + M_t = B_{t-1} + M_{t-1} + W_t N_t - T_t \tag{2}$$

- By letting $A_t \equiv B_{t-1} + M_{t-1}$ denote total financial wealth at the beginning of the period t, the flow budget constraint can be rewritten as

$$P_t C_t + Q_t A_{t+1} = A_{t-1} + W_t N_t - T_t \tag{3}$$

- Subject to a solvency constraint

$$\lim_{T \to \infty} E_t\{A_t\} \geq 0 \tag{4}$$

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- Using A_t it is assumed that all financial assets yield gross nominal return $Q^{-1}(= exp\{i_t\})$
- Households *purchase* the utility-yielding 'services' of money balances at a unit price $(1 - Q_t) = 1 - exp\{-i_t\} \simeq i_t$
- The implicit price of money services roughly corresponds to the nominal interest rate, which in turn is the opportunity cost of holding money

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Optimality Conditions

- Two of the optimality conditions are same as those for the cashless model

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} \tag{5}$$

$$Q_t = \beta E_t \left\{ \frac{U_{c,t+1}}{U_{c,t}} \frac{P_t}{P_{t+1}} \right\} \tag{6}$$

- And there is an additional optimality condition given by

$$\frac{U_{m,t}}{U_{c,t}} = 1 - exp\{-i_t\} \tag{7}$$

- For any statement about consequences of having money in the utility function , more precision is needed about the way money balances interact with other variables in yielding utility

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An Example with Separable Utility

- Assume following functional form

$$U(C_t, \frac{M_t}{P_t}, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} + \frac{(M_t/P_t)^{1-\nu}}{1-\nu} - \frac{N_t^{1+\varphi}}{1+\varphi} \quad (8)$$

- Given assumed separability neither $U_{c,t}$ nor $U_{n,t}$ depend on the level of real balances
- As a result

$$C_t^\sigma N_t^\varphi = \frac{W_t}{P_t} \quad (9)$$

$$Q_t = \beta E_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right\} \quad (10)$$

remain unchanged and so do their log-linear counterparts

$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\}) \quad (11)$$

$$w_t - p_t = \sigma c_t + \varphi n_t \quad (12)$$

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- However, introduction of money in utility function allows a money demand equation to be derived from the households' optimal behaviour
- Given the specification of utility, the new optimality condition can be rewritten as

$$\frac{M_t}{P_t} = C_t^{\sigma/\nu}\left(1 - exp\{-i_t\}^{-1/\nu}\right) \quad (13)$$

- And in approximate log-linear form as

$$m_t - p_t = \frac{\sigma}{\nu}c_t - \eta i_t \quad (14)$$

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- The particular case of $\nu = \sigma$ is an appealing one, because it implies a unit elasticity with respect to consumption
- This assumption yields a conventional linear demand for money

$$m_t - p_t = c_t - \eta i_t \quad (15)$$

$$= y_t - \eta i_t \quad (16)$$

  assuming that all output is consumed
- (15) determines the equilibrium values for inflation and other nominal variables whenever the description of monetary policy involves the quantity of money
- Otherwise (15) determines the quantity of money that the central bank will need to supply in order to support the nominal interest rate implied by the policy rule

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An Example with Nonseparable Utility

- Let now consider an economy in which period utility is given

$$U(C_t, \frac{M_t}{P_t}, N_t) = \frac{X_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \quad (19)$$

where X_t is a composite index of consumption and real balances

$$\begin{aligned} X_t &\equiv \left[(1-\vartheta) C_t^{1-\nu} + \vartheta \left(\frac{M_t}{P_t} \right)^{1-\nu} \right]^{\frac{1}{1-\nu}} \\ &\equiv C_t^{1-\vartheta} \left(\frac{M_t}{P_t} \right)^{\vartheta} \end{aligned}$$

for $\nu = 1$

- ν represents the (inverse) elasticity of substitution between consumption and real balances, and ϑ the relative weight of real balances in utility

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whereas marginal utility of labour is, as before, given by $U_{n,t} = -N_t^{\varphi}$

- Optimality conditions of the household's problem are now

$$\frac{W_t}{P_t} = N_t^{\varphi} X_t^{\nu-\sigma} C_t^{\nu} (1-\vartheta)^{-1} \quad (20)$$

$$Q_t = \beta E_t \left\{ \left(\frac{C_{t+1}}{C_t}\right)^{-\nu} \left(\frac{X_{t+1}}{X_t}\right)^{\nu-\sigma} \frac{P_t}{P_{t+1}} \right\} \quad (21)$$

$$\frac{M_t}{P_t} = C_t (1 - exp\{-i_t\})^{-1/\nu} \left(\frac{\vartheta}{1-\vartheta}\right)^{\frac{1}{\nu}} \quad (22)$$

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An Example with Nonseparable Utility

- Optimality conditions imply that monetary policy is no longer neutral
  - Real money balances influence labour supply as well as consumption
  - Whereas depend on the nominal interest rate
- Different paths of the interest rate have different implications for real balances, consumption and labour supply
- Formally, we start noticing that the implied money demand equation (22) has following log-linear form

$$m_t - p_t = c_t - \eta i_t \tag{23}$$

where $\eta \equiv \frac{1}{\nu(exp\{\bar{i}\}-1)}$

- Money demand semi-elasticity depends on the elasticity of substitution between real balances and consumption ν^{-1}

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- Log-linearization of 20 yields

$$w_t - p_t = \sigma c_t + \varphi n_t + (\nu - \sigma)(c_t - x_t) \tag{24}$$

- Log-linearizing expression fo X_t, combining with 22 and substituting for x_t above yields

$$w_t - p_t = \sigma c_t + \varphi n_t + \chi(\nu - \sigma)(c_t - (m_t - p_t)) \tag{25}$$

where $\chi = \frac{\vartheta^{\frac{1}{\nu}}(1-\beta)^{1-\frac{1}{\nu}}}{(1-\vartheta)^{\frac{1}{\nu}}\vartheta^{\frac{1}{\nu}}(1-\beta)^{1-\frac{1}{\nu}}}$

- Finally, substituting for $c_t - (m_t - p_t)$ from log-linear money demand yields

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where $\chi = \frac{\vartheta^{\frac{1}{\nu}}(1-\beta)^{1-\frac{1}{\nu}}}{(1-\vartheta)^{\frac{1}{\nu}}\vartheta^{\frac{1}{\nu}}(1-\beta)^{1-\frac{1}{\nu}}}$

- Finally, substituting for $c_t - (m_t - p_t)$ from log-linear money demand yields

$$w_t - p_t = \sigma c_t + \varphi n_t + \chi\eta(\nu - \sigma)i_t \tag{26}$$

An Example with Nonseparable Utility

- Let's define steady-state ratio of real balances to consumption $k_m \equiv \frac{\bar{M}/\bar{P}}{\bar{C}}$
- Then using money demand equation one gets $k_m = \left(\frac{\vartheta}{(1-\beta)(1-\vartheta)}\right)^{\frac{1}{\nu}}$ and $\chi = \frac{k_m(1-\beta)}{1+k_m(1-\beta)}$
- Multiplying χ, η and $\nu - \sigma$ one gets $\omega \equiv \frac{k_m\beta\left(1-\frac{\sigma}{\nu}\right)}{1+k_m(1-\beta)}$ and

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- Impact of interest rate on labour supply depends on the sign of ω and that is determined by the sign of $\nu - \sigma$
- When $\nu > \sigma$ (implying $\omega > 0$) the reduction in real balances (increase in interest rate) brings down marginal utility of consumption, lowering the quantity of labour supplied

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- Log-linear approximation of the Euler equation (21) looks as follows

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\begin{aligned}
c_t &= E_t\{c_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} \\
&- (\nu - \sigma)E_t\{(c_{t+1} - x_{t+1}) - (c_t - x_t)\} - \rho) \\
c_t &= E_t\{c_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} \\
&- \chi(\nu - \sigma)E_t\{\Delta c_{t+1} - \Delta(m_{t+1} - p_{t+1})\} - \rho) \\
c_t &= E_t\{c_{t+1}\} - \frac{1}{\sigma}\left(i_t - E_t\{\pi_{t+1}\} - \omega E_t\{\Delta i_{t+1}\} - \rho\right) \qquad (28)
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An Example with Nonseparable Utility - Equilibrium

- Let's start by combining labour supply and labour demand which both equal the real wage

$$\sigma c_t + \varphi n_t + \omega i_t = y_t - n_t + log(1 - \alpha) \tag{29}$$

- Then using market clearing condition $y_t = c_t$ and production function $y_t = a_t + \alpha n_t$ yields

$$y_t = \psi_{ya} a_t + \psi_{yi} i_t + \upsilon_{ya} \tag{30}$$

where $\psi_{yi} \equiv \frac{\omega(1-alpha)}{\sigma+\varphi+\alpha(1-\sigma)}$

- Equilibrium output is no more invariant to monetary policy, money is not neutral and the above equilibrium condition does not suffice to determine output

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- In order to pin down equilibrium path of endogenous variables equilibrium condition for output has to be combined with Euler equation and description of monetary policy

$$y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma}\left(i_t - E_t\{\pi_{t+1}\} - \omega E_t\{\Delta i_{t+1}\} - \rho\right) \quad (31)$$

$$i_t = \rho + \Phi_\pi \pi_t + \upsilon_t \quad (32)$$

where we assume that υ follows a stationary AR(1) process $\upsilon_t = \rho_\upsilon \upsilon_{t-1} + \epsilon_t^\upsilon$

- Similarly assume that the technology follows the AR(1) process $a_t = \rho_a a_{t-1} + \epsilon_t^a$

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An Example with Nonseparable Utility - Equilibrium

- Closed form expression for the equilibrium level of inflation, interest rate and output looks as follows

$$
\begin{aligned}
\pi_t &= -\frac{\sigma(1-\rho_a)\psi_{ya}}{\Phi_\pi(1+\omega\psi)(1-\Theta\rho_a)}a_t - \frac{1+(1-\rho_\upsilon)\omega\psi}{\Phi_\pi(1+\omega\psi)(1-\Theta\rho_\upsilon)}\upsilon_t \\
i_t &= -\frac{\sigma(1-\rho_a)\psi_{ya}}{(1+\omega\psi)(1-\Theta\rho_a)}a_t - \frac{\rho_\upsilon}{\Phi_\pi(1+\omega\psi)(1-\Theta\rho_\upsilon)}\upsilon_t \\
y_t &= \psi_{ya}\left(1+\frac{\sigma(1-\rho_a)\psi_{yi}}{(1+\omega\psi)(1-\Theta\rho_a)}\right)a_t + \frac{\rho_\upsilon\psi_{yi}}{\Phi_\pi(1+\omega\psi)(1-\Theta\rho_\upsilon)}\upsilon_t
\end{aligned}
$$

where $\Theta \equiv \frac{1+\omega\psi\Phi_\pi}{(1+\omega\psi)\Phi_\pi}$ and $\psi \equiv \frac{\alpha+\varphi}{\sigma(1-\alpha)+\alpha+\varphi}$

An Example with Nonseparable Utility - Impact of Monetary Policy

- Interest rate multiplier of output, conditional on an exogenous monetary policy shock, is given by $\frac{dy_t}{di_t} = \frac{dy_t/d\upsilon_t}{di_t/d\upsilon_t} = -\psi_{yi}$
- In order to get sense for the magnitude, recall that $\psi_{yi} \equiv \frac{\omega(1-alpha)}{\sigma+\varphi+\alpha(1-\sigma)}$. Assuming common calibration $\sigma = \varphi = 1$ and $\alpha = 1/3$, one gets $\psi_{yi} = 1/3\omega$
- Having in mind that ω itself depends mainly on k_m, especially when β is close to one and η is relatively small, one can approximate $\psi_{yi} = 1/3k_m$
- Size of k_m depends on the definition of money and ranges from $k_m \simeq 0.3$ to $k_m \simeq 3$ for monetary base and M2 respectively

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An Example with Nonseparable Utility - Impact of Monetary Policy

- Leading to values of ψ_{yi} in the range from 0.1 to 1 and impact of one percent increase in interest rate (annualized) in the range from 0.025 to 0.25 percent.
- The latter value, while small, appears to be closer to the estimated output effects of a monetary policy shock found in the literature
- However, there are other aspects of the transmission of monetary shocks that are at odds with the evidence.
- Note that

$$\frac{d\pi_t}{di_t} = \frac{d\pi_t/d\upsilon_t}{di_t/d\upsilon_t} = (1+(1-\rho_\upsilon)\omega\psi\rho_\upsilon^{-1} > 0$$

$$\frac{dr_t}{di_t} = 1 - \frac{dE_t\{\pi_{t+1}\}/d\upsilon_t}{di_t/d\upsilon_t} = -(1-\rho_\upsilon)\omega\psi < 0$$

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An Example with Nonseparable Utility - Long-run Implications

- Long-run output effects of monetary policy that permanently raises nominal interest rate is same as the short-run, i.e. $-\psi_{yi}$
- Any permanent increase in inflation and nominal interest rate leads to lower output
- In principle there is no problem with that, however, the lack of significant empirical relationship between long-run inflation and economic activity (at least at low levels of inflation), suggests a low value for k_m and ψ_{yi}
- Unfortunately, negligible long-run tradeoff is associated with negligible short run effects of monetary policy

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Optimal Monetary Policy

- Hypothetical social planner seeking to maximize the utility of representative household would solve a sequence of static problems of the form

$$maxU(C_t, \frac{M_t}{P_t}, N_t)$$

subject to the resource constraint

$$C_t = A_t N_t^{1-\alpha}$$

The optimality conditions for that problem are given

$$-\frac{U_{n,t}}{U_{c,t}} = (1-\alpha) A_t N_t^{-\alpha} \quad (33)$$

$$U_{m,t} = 0 \quad (34)$$

the latter condition equates marginal utility of real balances to the social marginal cost of their production, which is implicitly assumed to be zero

- As household's optimal choice of money balances requires

$$-\frac{U_{m,t}}{U_{c,t}} = (1 - exp\{-i_t\})$$

efficiency condition (34 will be satisfied, if and only if, $i_t = 0$ for all t, a policy known as the *Friedman rule*

- Note that such a policy implies an average rate of inflation

$$\pi = -\rho < 0$$

i.e., prices will decline on average at the rate of time preference

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Optimal Monetary Policy

- A policy rule of the form $i_t = 0$ for all t leads to price level indeterminacy, so the central bank cannot just set $i_t = 0$ for all t in order to implement the *Friedman rule*
- By following a rule of the form

$$i_t = \Phi(r_{t-1} - \pi_t)$$

  the central bank can avoid price level indeterminacy, while setting i_t on average equal to zero
- To see this, combine the above rule with Fisher equation $i_t = r_t + E_t\{\pi_{t+1}\}$, which implies the difference equation

$$E_t\{i_{t+1}\} = \Phi i_t \qquad (35)$$

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- Under the above rule inflation is fully predictable and given by

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