A Classical Monetary Model - Money in the Utility Function

Jarek Hurnik

Department of Economics

Lecture III
So far, the only role played by money was to serve as a numéraire, i.e. we dealt with a *cashless economy*. We incorporate a role of money other than that of unit of account to generate demand for money. We assume that real balances are an argument in the utility function. Be aware that there are other options such as cash-in-advance.
Basic Facts

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  - Be aware that there are other options such as cash-in-advance
Households

- Households’ preferences are now given by

\[
E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, \frac{M_t}{P_t}, N_t)
\]  

(1)

- Period utility is increasing and concave in real balances \( \frac{M_t}{P_t} \)

- And the flow budget constraint takes the form

\[
P_tC_t + Q_tB_t + M_t = B_{t-1} + M_{t-1} + W_tN_t - T_t
\]  

(2)

- By letting \( A_t \equiv B_{t-1} + M_{t-1} \) denote total financial wealth at the beginning of the period \( t \), the flow budget constraint can be rewritten as

\[
P_tC_t + Q_tA_{t+1} = A_{t-1} + W_tN_t - T_t
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(3)

- Subject to a solvency constraint

\[
\lim_{T \to \infty} E_T\{A_t\} \geq 0
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Using $A_t$ it is assumed that all financial assets yield gross nominal return $Q^{-1}(= \exp\{i_t\})$

Households *purchase* the utility-yielding ’services’ of money balances at a unit price $(1 - Q_t) = 1 - \exp\{-i_t\} \simeq i_t$

The implicit price of money services roughly corresponds to the nominal interest rate, which in turn is the opportunity cost of holding money.
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The implicit price of money services roughly corresponds to the nominal interest rate, which in turn is the opportunity cost of holding money.
Two of the optimality conditions are same as those for the cashless model

\[
- \frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} \tag{5}
\]

\[
Q_t = \beta E_t \left\{ \frac{U_{c,t+1}}{U_{c,t}} \frac{P_t}{P_{t+1}} \right\} \tag{6}
\]

And there is an additional optimality condition given by

\[
\frac{U_{m,t}}{U_{c,t}} = 1 - \exp\{-i_t\} \tag{7}
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For any statement about consequences of having money in the utility function, more precision is needed about the way money balances interact with other variables in yielding utility.
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An Example with Separable Utility

Assume following functional form

\[ U(C_t, \frac{M_t}{P_t}, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} + \frac{(M_t/P_t)^{1-\nu}}{1-\nu} - \frac{N_t^{1+\varphi}}{1+\varphi} \]  

(8)

Given assumed separability neither \( U_{c,t} \) nor \( U_{n,t} \) depend on the level of real balances

As a result

\[ C_t^\sigma N_t^\varphi = \frac{W_t}{P_t} \]  

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\[ Q_t = \beta E_t \left\{ \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right\} \]  

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remain unchanged and so do their log-linear counterparts

\[ c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma} (i_t - E_t\{\pi_{t+1}\}) \]  

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\[ w_t - p_t = \sigma c_t + \varphi n_t \]  

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- However, introduction of money in utility function allows a money demand equation to be derived from the households’ optimal behaviour.

- Given the specification of utility, the new optimality condition can be rewritten as:

\[
\frac{M_t}{P_t} = C_t^{\sigma/\nu} \left( 1 - \exp\{-i_t\}^{-1/\nu} \right)
\]  

(13)

- And in approximate log-linear form as:

\[
m_t - p_t = \frac{\sigma}{\nu} c_t - \eta i_t
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(14)

where \( \eta \equiv \frac{1}{\nu(\exp\{i\} - 1)} \approx \frac{1}{\nu i} \).
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An Example with Separable Utility

- The particular case of $\nu = \sigma$ is an appealing one, because it implies a unit elasticity with respect to consumption.
- This assumption yields a conventional linear demand for money.

\[
m_t - p_t = c_t - \eta i_t \quad (15)
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= y_t - \eta i_t \quad (16)
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assuming that all output is consumed.

- (15) determines the equilibrium values for inflation and other nominal variables whenever the description of monetary policy involves the quantity of money.
- Otherwise (15) determines the quantity of money that the central bank will need to supply in order to support the nominal interest rate implied by the policy rule.
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An Example with Nonseparable Utility

Let now consider an economy in which period utility is given

$$U(C_t, \frac{M_t}{P_t}, N_t) = \frac{X_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}$$

(19)

where $X_t$ is a composite index of consumption and real balances

$$X_t \equiv \left[ (1-\psi) C_t^{1-\nu} + \psi \left(\frac{M_t}{P_t}\right)^{1-\nu} \right]^{\frac{1}{1-\nu}}$$

$$\equiv C_t^{1-\psi} \left(\frac{M_t}{P_t}\right)^{\psi}$$

for $\nu = 1$

$\psi$ represents the (inverse) elasticity of substitution between consumption and real balances, and $\psi$ the relative weight of real balances in utility.
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U_{c,t} = (1 - \vartheta) X_t^{\nu-\sigma} C_t^{-\nu}
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whereas marginal utility of labour is, as before, given by \( U_{n,t} = -N_t^{\varphi} \)

- Optimality conditions of the household’s problem are now

\[
\frac{W_t}{P_t} = N_t^{\varphi} X_t^{\nu-\sigma} C_t^{\nu} (1 - \vartheta)^{-1}
\]

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\[
Q_t = \beta E_t \left\{ \left( \frac{C_{t+1}}{C_t} \right)^{-\nu} \left( \frac{X_{t+1}}{X_t} \right)^{\nu-\sigma} \frac{P_t}{P_{t+1}} \right\}
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\frac{M_t}{P_t} = C_t (1 - \exp\{-i_t\})^{-1/\nu} \left( \frac{\vartheta}{1 - \vartheta} \right)^{\frac{1}{\nu}}
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An Example with Nonseparable Utility

- Optimality conditions imply that monetary policy is no longer neutral
  - Real money balances influence labour supply as well as consumption
  - Whereas depend on the nominal interest rate
- Different paths of the interest rate have different implications for real balances, consumption and labour supply
- Formally, we start noticing that the implied money demand equation (22) has following log-linear form

\[ m_t - p_t = c_t - \eta i_t \]  

(23)

where \( \eta \equiv \frac{1}{\nu (\exp \{i\} - 1)} \)

- Money demand semi-elasticity depends on the elasticity of substitution between real balances and consumption \( \nu^{-1} \)
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An Example with Nonseparable Utility

- Log-linearization of 20 yields

\[ w_t - p_t = \sigma c_t + \varphi n_t + (\nu - \sigma)(c_t - x_t) \]  \hspace{1cm} (24)

- Log-linearizing expression for \(X_t\), combining with 22 and substituting for \(x_t\) above yields

\[ w_t - p_t = \sigma c_t + \varphi n_t + \chi(\nu - \sigma)(c_t - (m_t - p_t)) \]  \hspace{1cm} (25)

where \( \chi = \) 

\[
\frac{\vartheta^{\frac{1}{\nu}} (1 - \beta)^{\frac{1}{\nu}}}{(1 - \vartheta)^{\frac{1}{\nu}} \vartheta^{\frac{1}{\nu}} (1 - \beta)^{\frac{1}{\nu} - \frac{1}{\nu}}}
\]

- Finally, substituting for \(c_t - (m_t - p_t)\) from log-linear money demand yields

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An Example with Nonseparable Utility

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where \( \chi = \frac{\vartheta \frac{1}{\nu} (1-\beta)^{1-\frac{1}{\nu}}}{(1-\vartheta) \frac{1}{\nu} \vartheta \frac{1}{\nu} (1-\beta)^{1-\frac{1}{\nu}}} \)

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- Let’s define steady-state ratio of real balances to consumption \( k_m \equiv \frac{\bar{M}}{\bar{P} C} \)

- Then using money demand equation one gets \( k_m = \left( \frac{\vartheta}{(1-\beta)(1-\vartheta)} \right)^{\frac{1}{\nu}} \) and

\[
\chi = \frac{k_m(1-\beta)}{1+k_m(1-\beta)}
\]

- Multiplying \( \chi, \eta \) and \( \nu - \sigma \) one gets \( \omega \equiv \frac{k_m \beta (1-\frac{\sigma}{\nu})}{1+k_m(1-\beta)} \) and

\[
\omega_t - p_t = \sigma c_t + \varphi n_t + \omega i_t \tag{27}
\]

- Impact of interest rate on labour supply depends on the sign of \( \omega \) and that is determined by the sign of \( \nu - \sigma \)

- When \( \nu > \sigma \) (implying \( \omega > 0 \)) the reduction in real balances (increase in interest rate) brings down marginal utility of consumption, lowering the quantity of labour supplied
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An Example with Nonseparable Utility

- Log-linear approximation of the Euler equation (21) looks as follows

\[
    c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma} (i_t - E_t\{\pi_{t+1}\}) \\
    - (\nu - \sigma)E_t\{(c_{t+1} - x_{t+1}) - (c_t - x_t)\} - \rho
\]

\[
    c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma} (i_t - E_t\{\pi_{t+1}\}) \\
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- Anticipation of a nominal interest rate increase lowers the expected level of the marginal utility of consumption and induces an increase in current consumption
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Anticipation of a nominal interest rate increase lowers the expected level of the marginal utility of consumption and induces an increase in current consumption.
Let’s start by combining labour supply and labour demand which both equal the real wage

\[ \sigma c_t + \varphi n_t + \omega i_t = y_t - n_t + \log(1 - \alpha) \]  

Then using market clearing condition \( y_t = c_t \) and production function \( y_t = a_t + \alpha n_t \) yields

\[ y_t = \psi_y a_t + \psi_y i_t + \nu_y \]  

where \( \psi_y \equiv \frac{\omega(1-\alpha)}{\sigma + \varphi + \alpha(1-\sigma)} \)

Equilibrium output is no more invariant to monetary policy, money is not neutral and the above equilibrium condition does not suffice to determine output.
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An Example with Nonseparable Utility - Equilibrium

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- Then using market clearing condition \( y_t = c_t \) and production function \( y_t = a_t + \alpha n_t \) yields

\[ y_t = \psi_{ya} a_t + \psi_{yi} i_t + \nu_{ya} \]  

(30)

where \( \psi_{yi} \equiv \frac{\omega(1-\text{alpha})}{\sigma + \varphi + \alpha(1-\sigma)} \)

- Equilibrium output is no more invariant to monetary policy, money is not neutral and the above equilibrium condition does not suffice to determine output
In order to pin down equilibrium path of endogenous variables
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\[ y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma} (i_t - E_t\{\pi_{t+1}\} - \omega E_t\{\Delta i_{t+1}\} - \rho) \]  \hspace{1cm} (31)

\[ i_t = \rho + \Phi \pi \pi_t + u_t \]  \hspace{1cm} (32)

where we assume that \( u \) follows a stationary AR(1) process

\[ u_t = \rho u u_{t-1} + \epsilon^u_t \]

Similarly assume that the technology follows the AR(1) process

\[ a_t = \rho_a a_{t-1} + \epsilon^a_t \]
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where we assume that \( v \) follows a stationary AR(1) process

\[ v_t = \rho_v v_{t-1} + \epsilon_v^t \]

Similarly assume that the technology follows the AR(1) process

\[ a_t = \rho_a a_{t-1} + \epsilon_a^t \]
An Example with Nonseparable Utility - Equilibrium

- Closed form expression for the equilibrium level of inflation, interest rate and output looks as follows

\[
\pi_t = -\frac{\sigma(1 - \rho_a)\psi_ya}{\Phi_\pi(1 + \omega\psi)(1 - \Theta\rho_a)} a_t - \frac{1 + (1 - \rho_v)\omega\psi}{\Phi_\pi(1 + \omega\psi)(1 - \Theta\rho_v)} \upsilon_t
\]

\[
i_t = -\frac{\sigma(1 - \rho_a)\psi_ya}{(1 + \omega\psi)(1 - \Theta\rho_a)} a_t - \frac{\rho_v}{\Phi_\pi(1 + \omega\psi)(1 - \Theta\rho_v)} \upsilon_t
\]

\[
y_t = \psi_ya \left(1 + \frac{\sigma(1 - \rho_a)\psi_yi}{(1 + \omega\psi)(1 - \Theta\rho_a)}\right) a_t + \frac{\rho_v\psi_yi}{\Phi_\pi(1 + \omega\psi)(1 - \Theta\rho_v)} \upsilon_t
\]

where \( \Theta \equiv \frac{1 + \omega\psi\Phi_\pi}{(1 + \omega\psi)\Phi_\pi} \) and \( \psi \equiv \frac{\alpha + \varphi}{\sigma(1 - \alpha) + \alpha + \varphi} \)
Interest rate multiplier of output, conditional on an exogenous monetary policy shock, is given by
\[ \frac{dy_t}{di_t} = \frac{dy_t/d\nu_t}{di_t/d\nu_t} = -\psi_{yi} \]

In order to get sense for the magnitude, recall that
\[ \psi_{yi} \equiv \frac{\omega(1-\alpha)}{\sigma + \varphi + \alpha(1-\sigma)}. \]

Assuming common calibration \( \sigma = \varphi = 1 \) and \( \alpha = 1/3 \), one gets \( \psi_{yi} = 1/3\omega \).

Having in mind that \( \omega \) itself depends mainly on \( k_m \), especially when \( \beta \) is close to one and \( \eta \) is relatively small, one can approximate \( \psi_{yi} = 1/3k_m \).

Size of \( k_m \) depends on the definition of money and ranges from \( k_m \simeq 0.3 \) to \( k_m \simeq 3 \) for monetary base and M2 respectively.
An Example with Nonseparable Utility - Impact of Monetary Policy

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An Example with Nonseparable Utility - Impact of Monetary Policy

- Leading to values of $\psi_{yi}$ in the range from 0.1 to 1 and impact of one percent increase in interest rate (annualized) in the range from 0.025 to 0.25 percent.

- The latter value, while small, appears to be closer to the estimated output effects of a monetary policy shock found in the literature.

- However, there are other aspects of the transmission of monetary shocks that are at odds with the evidence.

- Note that

$$\frac{d\pi_t}{di_t} = \frac{d\pi_t/d\nu_t}{di_t/d\nu_t} = (1 + (1 - \rho_v)\omega\psi\rho_v^{-1}) > 0$$

$$\frac{dr_t}{di_t} = 1 - \frac{dE_t\{\pi_{t+1}\}/d\nu_t}{di_t/d\nu_t} = -(1 - \rho_v)\omega\psi < 0$$
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An Example with Nonseparable Utility - Long-run Implications

- Long-run output effects of monetary policy that permanently raises nominal interest rate is same as the short-run, i.e. $-\psi y_i$
- Any permanent increase in inflation and nominal interest rate leads to lower output
- In principle there is no problem with that, however, the lack of significant empirical relationship between long-run inflation and economic activity (at least at low levels of inflation), suggests a low value for $k_m$ and $\psi y_i$
- Unfortunately, negligible long-run tradeoff is associated with negligible short run effects of monetary policy
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Optimal Monetary Policy

- Hypothetical social planner seeking to maximize the utility of representative household would solve a sequence of static problems of the form

$$\max U(C_t, \frac{M_t}{P_t}, N_t)$$

subject to the resource constraint

$$C_t = A_t N_t^{1-\alpha}$$

The optimality conditions for that problem are given

$$-\frac{U_{n,t}}{U_{c,t}} = (1 - \alpha)A_t N_t^{-\alpha}$$  (33)

$$U_{m,t} = 0$$  (34)

the latter condition equates marginal utility of real balances to the social marginal cost of their production, which is implicitly assumed to be zero
As household’s optimal choice of money balances requires:

\[-\frac{U_{m,t}}{U_{c,t}} = (1 - \exp\{-i_t\})\]

efficiency condition (34 will be satisfied, if and only if, $i_t = 0$ for all $t$, a policy known as the *Friedman rule*).

Note that such a policy implies an average rate of inflation

\[\pi = -\rho < 0\]

i.e., prices will decline on average at the rate of time preference.
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- A policy rule of the form \( i_t = 0 \) for all \( t \) leads to price level indeterminacy, so the central bank cannot just set \( i_t = 0 \) for all \( t \) in order to implement the *Friedman rule*.

- By following a rule of the form

  \[
  i_t = \Phi(r_{t-1} - \pi_t)
  \]

  the central bank can avoid price level indeterminacy, while setting \( i_t \) on average equal to zero.

- To see this, combine the above rule with Fisher equation

  \[
  i_t = r_t + E_t\{\pi_{t+1}\}, \text{ which implies the difference equation}
  \]

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- Under the above rule inflation is fully predictable and given by

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