L3. Overlapping Generations Model

Jarek Hurník

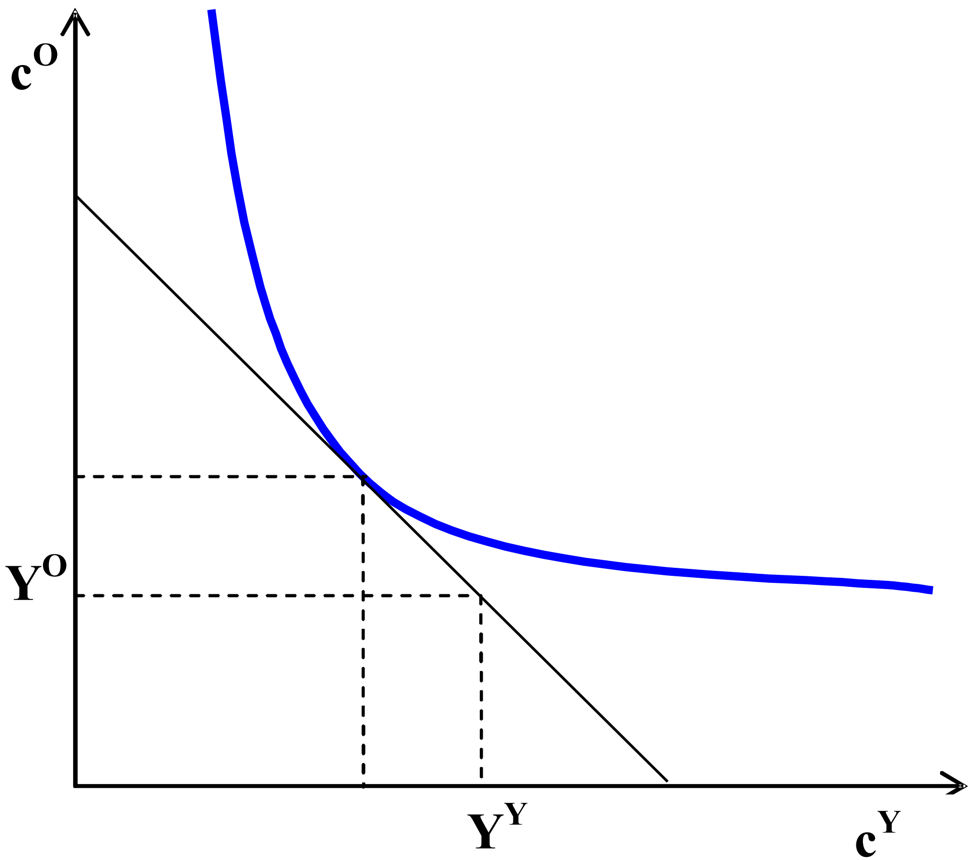
Assumptions

- Economy exists forever, but people live for two periods (Y,O) only;
- Single tradable good (real ER always equal to 1);
- Prices fully flexible;
- Income of a representative household Y_t^Y, Y_t^O falls down from heaven;
- No assets or debt when a person is born.

$$MaxU\left(c_t^Y; c_{t+1}^0\right) = \ln\left(c_t^Y\right) + \beta \ln\left(c_{t+1}^O\right)$$

$$s.t \quad c_t^Y + \frac{c_{t+1}^O}{1+r^*} = y_t^Y - \tau_t^Y + \frac{y_{t+1}^O - \tau_{t+1}^O}{1+r^*}$$

Euler Equation Again…



- Impatient people consume more when young and less when old;
- Consumption depends on life-time disposable income.

$$c_{t+1}^{O} = \left(1 + r^{*}\right)\beta\, c_{t}^{Y}$$

Consumption Level

$$c_t^Y + \frac{(1+r^*)\beta c_t^Y}{1+r^*} = y_t^Y - \tau_t^Y + \frac{y_{t+1}^O - \tau_{t+1}^O}{1+r^*}$$

$$c_t^Y = \frac{1}{1+\beta}\left[y_t^Y - \tau_t^Y + \frac{y_{t+1}^O - \tau_{t+1}^O}{1+r^*} \right]$$

$$c_{t+1}^O = \frac{(1+r^*)\beta}{1+\beta}\left[y_t^Y - \tau_t^Y + \frac{y_{t+1}^O - \tau_{t+1}^O}{1+r^*} \right]$$

ass. $N_t = N_{t+1} = 1$; y^Y, y^O, τ^Y, τ^O constant

$$C = \frac{1+(1+r^*)\beta}{1+\beta}\left[y^Y - \tau^Y + \frac{y^O - \tau^O}{1+r^*} \right]$$

Example – consumption level

$$y^Y = 100; \quad y^O = 100; \quad \beta = \frac{1}{1.07}; \; r^* = 0.05$$

$$c^Y = \frac{1}{1+\beta}\left[y^Y + \frac{y^O}{\left(1+r^*\right)}\right] = \frac{1.07}{2.07}\left[100 + \frac{100}{1.05}\right] \cong 100.92$$

$$c^O = \left(1+r^*\right)\beta c^Y \cong \frac{1.05}{1.07}100.92 \cong 99.03$$

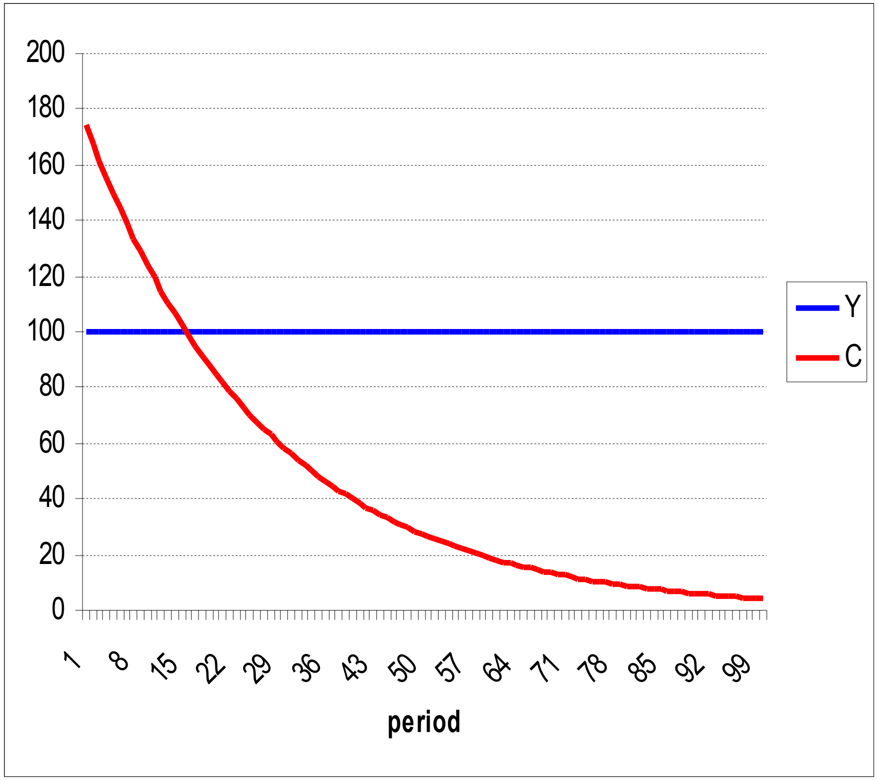
$$C \cong 100.92 + 99.03 \cong 199.95$$

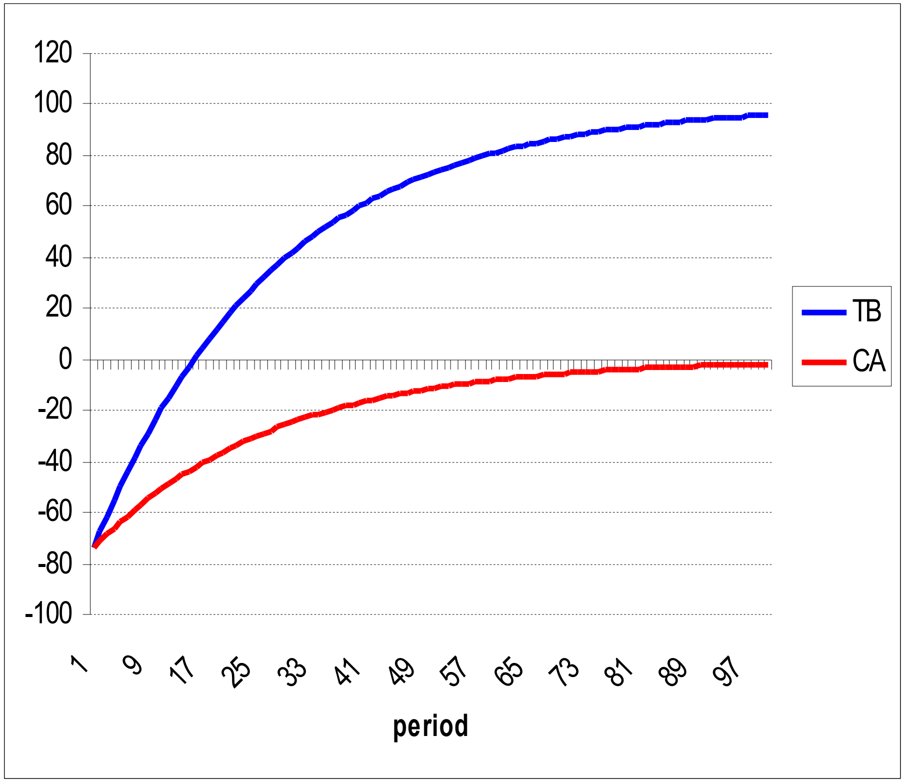
$$NX \cong 200 - 199.95 \cong 0.05$$

- Result much different compared to the infinite horizon.
- What works as the borrowing constraint?

Remember infinite horizon – Impatient People

$$Y = 100; \; I, G = 0; \; \beta = \frac{1}{1.07}; \; r^* = 0.05; \; \sigma = 2; \; B_1 = 0$$





Government

$$B_{t+1}^{G} - B_{t}^{G} = \tau_{t}^{Y} + \tau_{t}^{O} + r^{*}B_{t}^{G} - G_{t}$$

$$\sum_{s=t}^{\infty}\left(\frac{1}{1+r^{*}}\right)^{s-t}G_{s} = \left(1+r^{*}\right)B_{t}^{G} + \sum_{s=t}^{\infty}\left(\frac{1}{1+r^{*}}\right)^{s-t}\left(\tau_{s}^{Y} + \tau_{s}^{O}\right)$$

ass. $N_{t} = N_{t+1} = 1; y^{Y}, y^{O}, \tau^{Y}, \tau^{O}, G$ constant

$$G = r^{*}B^{G} + \tau_{s}^{Y} + \tau_{s}^{O}$$

$$C = \frac{1+\left(1+r^{*}\right)\beta}{1+\beta}\left[y^{Y} - \tau^{Y} + \frac{y^{O} - \tau^{O}}{1+r^{*}}\right]$$

$$C = \frac{1+\left(1+r^{*}\right)\beta}{1+\beta}\left[y^{Y} + \frac{y^{O} - G - r^{*}\tau^{Y} + r^{*}B^{G}}{1+r^{*}}\right]$$

„Twin Deficits"
(Ricardian equivalence)

$$y_t^Y = 100; \quad y_t^O = 100; \quad \beta = \frac{1}{1.05}; \; r^* = 0.05$$

$$\tau_0^Y = \tau_0^O = -10; \; \tau_{1,2,\ldots}^Y = \tau_{1,2,\ldots}^O = r^*\left(\frac{\tau_0^Y + \tau_0^O}{2}\right) = 0.5; \quad G = 0; \; B_0^{\mathrm{G}} = 0$$

$$c_0^O = \frac{\left(1+r^*\right)\beta}{1+\beta}\left[y_{-1}^Y + \frac{y_0^O}{\left(1+r^*\right)}\right] - \tau_0^O = \frac{1.05}{2.05}\left[100 + \frac{100}{1.05}\right] + 10 = 110$$

$$c_0^Y = \frac{1}{1+\beta}\left[y_0^Y - \tau_0^Y + \frac{y_1^O - \tau_1^Y}{\left(1+r^*\right)}\right] = \frac{1.05}{2.05}\left[110 + \frac{99.5}{1.05}\right] \cong 104.88$$

$$TB_0 = CA_0 = y_0^Y + y_0^O - c_0^Y - c_0^O \cong 200 - 214.88 \cong -14.88$$

$$c_1^O = \frac{(1+r^*)\beta}{1+\beta}\left[y_0^Y - \tau_0^Y + \frac{y_1^O - \tau_1^O}{(1+r^*)}\right] = \frac{1.05}{2.05}\left[110 + \frac{99.5}{1.05}\right] \cong 104.88$$

$$c_1^Y = \frac{1}{1+\beta}\left[y_1^Y - \tau_1^Y + \frac{y_2^O - \tau_2^O}{(1+r^*)}\right] = \frac{1.05}{2.05}\left[99.5 + \frac{99.5}{1.05}\right] = 99.5$$

$$TB_1 = y_1^Y + y_1^O - c_1^Y - c_1^O \cong 200 - 204.38 \cong -4.38$$

$$CA_1 = TB_1 + r^* CA_0 = -5.124$$

$$c_2^O = \frac{(1+r^*)\beta}{1+\beta}\left[y_1^Y - \tau_1^Y + \frac{y_2^O - \tau_2^O}{(1+r^*)}\right] = \frac{1.05}{2.05}\left[99.5 + \frac{99.5}{1.05}\right] = 99.5$$

$$c_2^Y = \frac{1}{1+\beta}\left[y_2^Y - \tau_2^Y + \frac{y_3^O - \tau_3^O}{(1+r^*)}\right] = \frac{1.05}{2.05}\left[99.5 + \frac{99.5}{1.05}\right] = 99.5$$

$$TB_2 = y_2^Y + y_2^O - c_2^Y - c_2^O \cong 200 - 199 = 1$$

$$CA_2 = TB_2 + r^*(CA_0 + CA_1) = 1 + 0.05 * (-20) = 0$$

Empirical Evidence

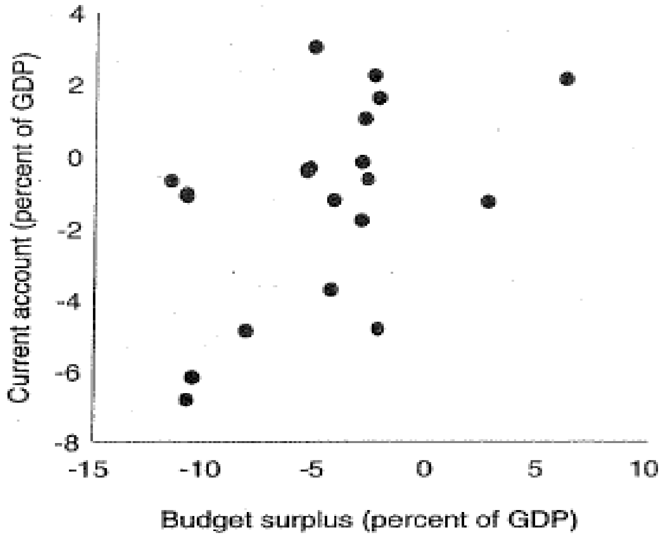


Figure 3.1
Current accounts and fiscal surpluses of industrial countries, 1981–86

$$CA/Y = -3.55 + 0.78(T - G)/Y, \qquad R^2 = 0.24.$$
$$\quad (4.06) \quad (0.33)$$

- There is some evidence of „twin deficits“ for 1981-86, both for the US and industrial countries in general.
- It worked for German re-unification, too.
- What about the US now ???

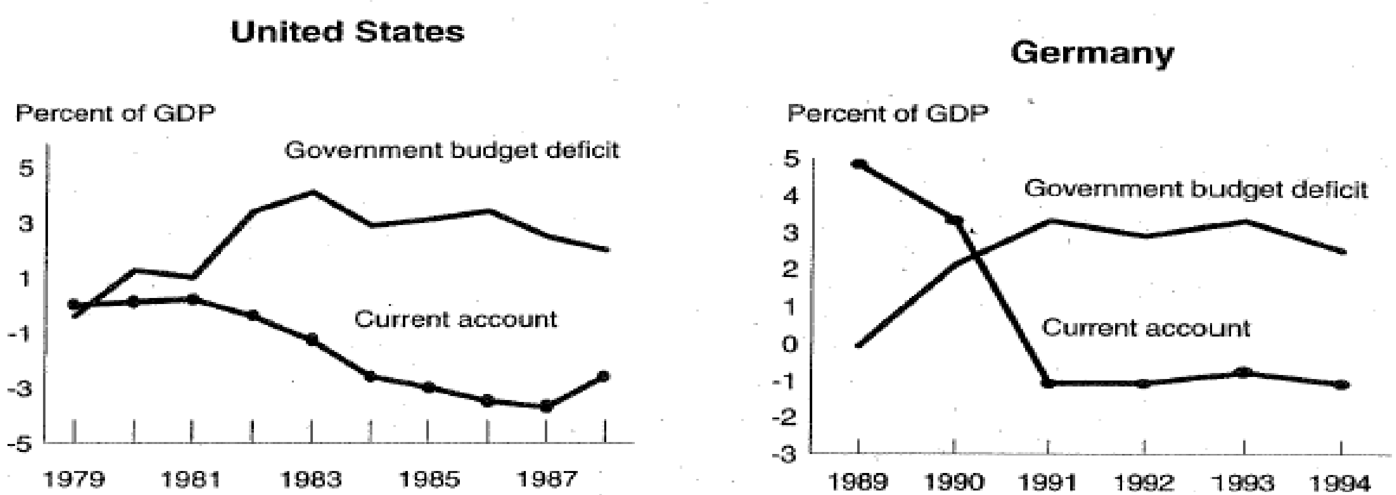


Figure 3.2
Government and foreign borrowing: United States and Germany

Savings and CA

$$CA_t = \left(B_{t+1} - B_t\right) = \left(B_{t+1}^P - B_t^P\right) + \left(B_{t+1}^G - B_t^G\right)$$

$$S_t^Y = B_{t+1}^P; \quad S_t^O = -S_{t-1}^Y = -B_t^P$$

$$S_t^P = S_t^Y + S_t^O = B_{t+1}^P - B_t^P$$

$$B_{t+1} = B_{t+1}^P + B_{t+1}^G = S_t^Y + B_{t+1}^G$$

$$\text{ass.} \left(1 + r^*\right)\beta = 1$$

$$S_t^Y = y_t^Y - \tau_t^Y - c_t^Y = y_t^Y - \tau_t^Y - \frac{1}{1+\beta}\left[y_t^Y - \tau_t^Y + \frac{y_{t+1}^O - \tau_{t+1}^O}{1+r^*}\right]$$

$$S_t^Y = \frac{\beta}{1+\beta}\left[\left(y_t^Y - \tau_t^Y\right) - \left(y_{t+1}^O - \tau_{t+1}^O\right)\right] = B_{t+1}^P$$

$$S_t^P = S_t^Y - S_{t-1}^Y = \frac{\beta}{1+\beta}\left[\Delta\left(y_t^Y - \tau_t^Y\right) - \Delta\left(y_{t+1}^O - \tau_{t+1}^O\right)\right]$$

Savings and Growth

$$y_{t+1}^{O} = (1+e)y_t^{Y}; \quad y_{t+1}^{Y} = (1+g)y_t^{Y}$$

$$S_t^{Y} = \frac{\beta}{1+\beta}\left[y_t^{Y} - y_{t+1}^{O}\right] = \frac{\beta}{1+\beta}\left[y_t^{Y} - (1+e)y_t^{Y}\right] = \frac{\beta}{1+\beta}(-e)y_t^{Y}$$

$$S_t^{O} = -S_{t-1}^{Y} = \frac{\beta}{1+\beta}e\frac{y_t^{Y}}{1+g}$$

$$S_t^{P} = \frac{\beta}{1+\beta}\frac{-eg}{1+g}y_t^{Y}$$

$$Y_t = y_t^{Y} + y_t^{O} = y_t^{Y} + \frac{1+e}{1+g}y_t^{Y} = \frac{2+g+e}{1+g}y_t^{Y}$$

$$\frac{S_t^{P}}{Y_t} = -\frac{\beta}{1+\beta}\frac{eg}{2+e+g} \quad \Rightarrow \quad \frac{\partial\left(S_t^{P}/Y_t\right)}{\partial g} = -\frac{\beta}{1+\beta}\frac{e(2+e)}{(2+e+g)^2}$$

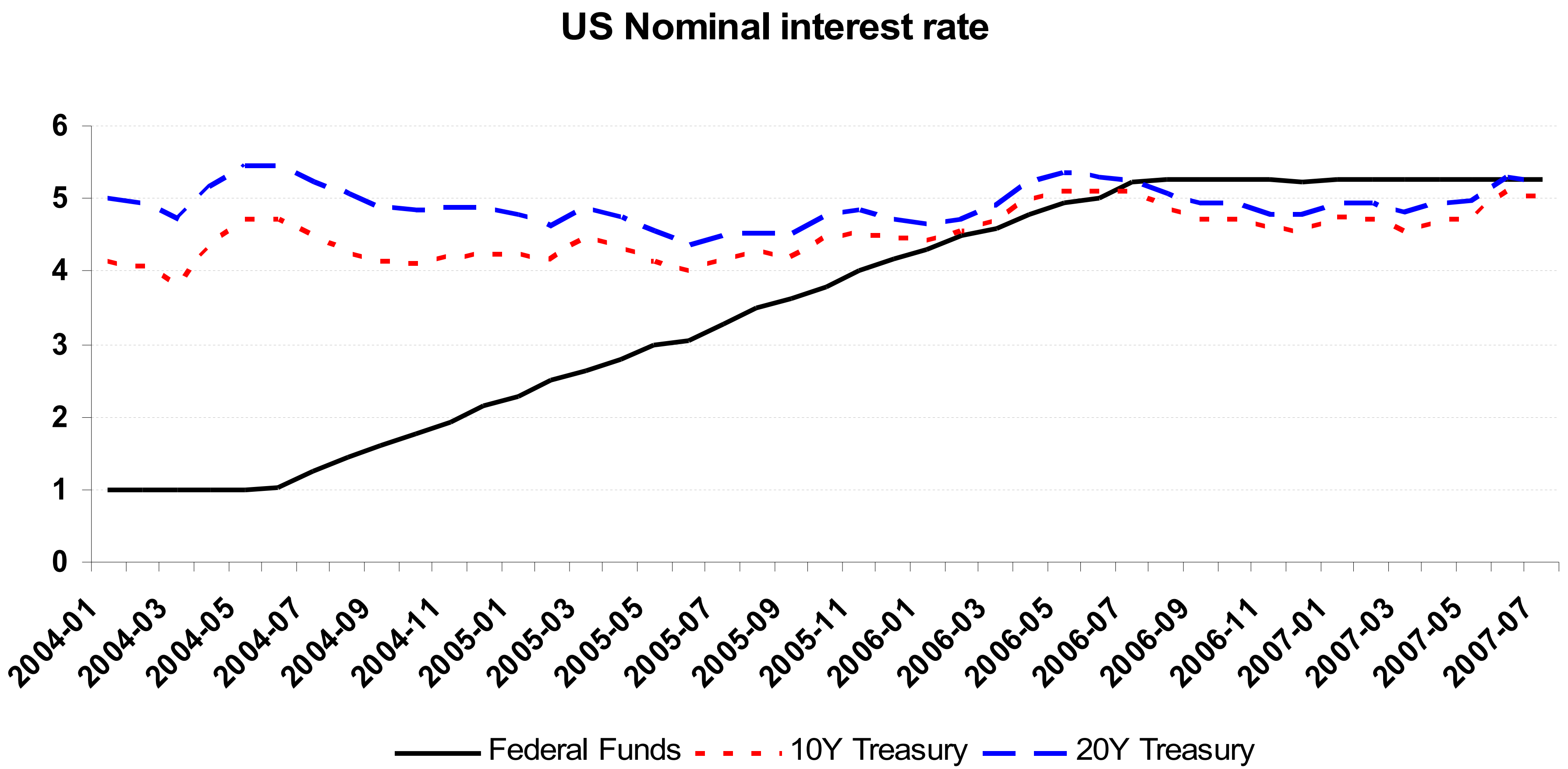
$$\frac{S_t^{P}}{Y_t} = \frac{(N_t - N_{t-1})S_t^{Y}}{N_t y_t^{Y} + N_{t-1}y_t^{O}} = \frac{nS_t^{Y}}{(1+n)y_t^{Y} + y_t^{O}} \quad \Rightarrow \quad \frac{\partial\left(S_t^{P}/Y_t\right)}{\partial n} = = \frac{S_t^{Y}(y_t^{Y} + y_t^{O})}{\left[(1+n)y_t^{Y} + y_t^{O}\right]^2}$$

Global Savings/Investment Imbalances

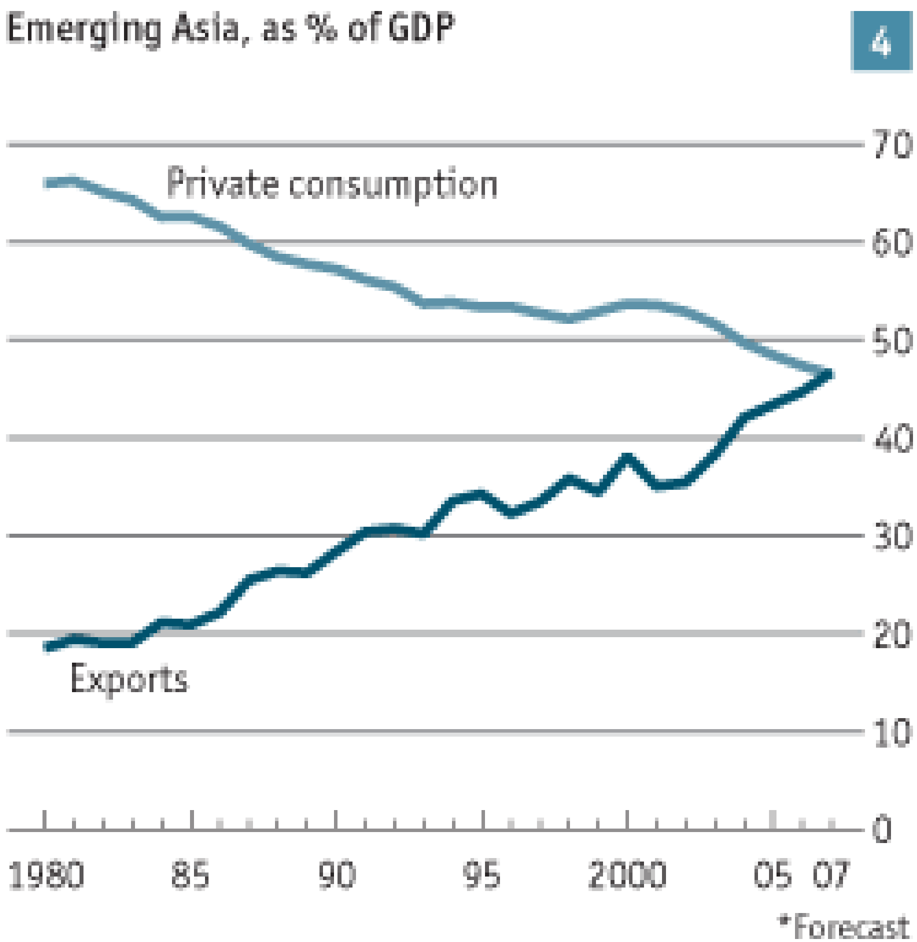
Global savings/investment imbalances

- Alan Greenspan's "conundrum" (2005 Testimony to the US Congress)
  - When the Federal Reserve started to increase the federal fund rate in 2004 the long-term bond rates did not increase but declined further
  - " … long-term interest rates have trended lower in recent months even as the Federal Reserve has raised the level of the target federal funds rate by 150 basis points. This development contrasts with most experience, which suggests that, other things being equal, increasing short-term interest rates are normally accompanied by a rise in longer-term yields."

Alan Greenspan's "conundrum"



Bernanke's "savings glut" (2005)



- Asian countries (but not only them) moved from current account deficits to surpluses
  - Reaction to the Asian crises in 1997
- Accumulation of foreign reserves
- Substantial decline in consumption/GDP ratios

Global imbalances are seen as the main cause of the recent bubble (Portes, 2009)

- Inflow of capital to countries with the most developed financial markets
  - Capital flowing ‘uphill’
- Financial sector’s response
  - Search for yield
  - Financial engineering
- Easy monetary conditions

John Taylor view

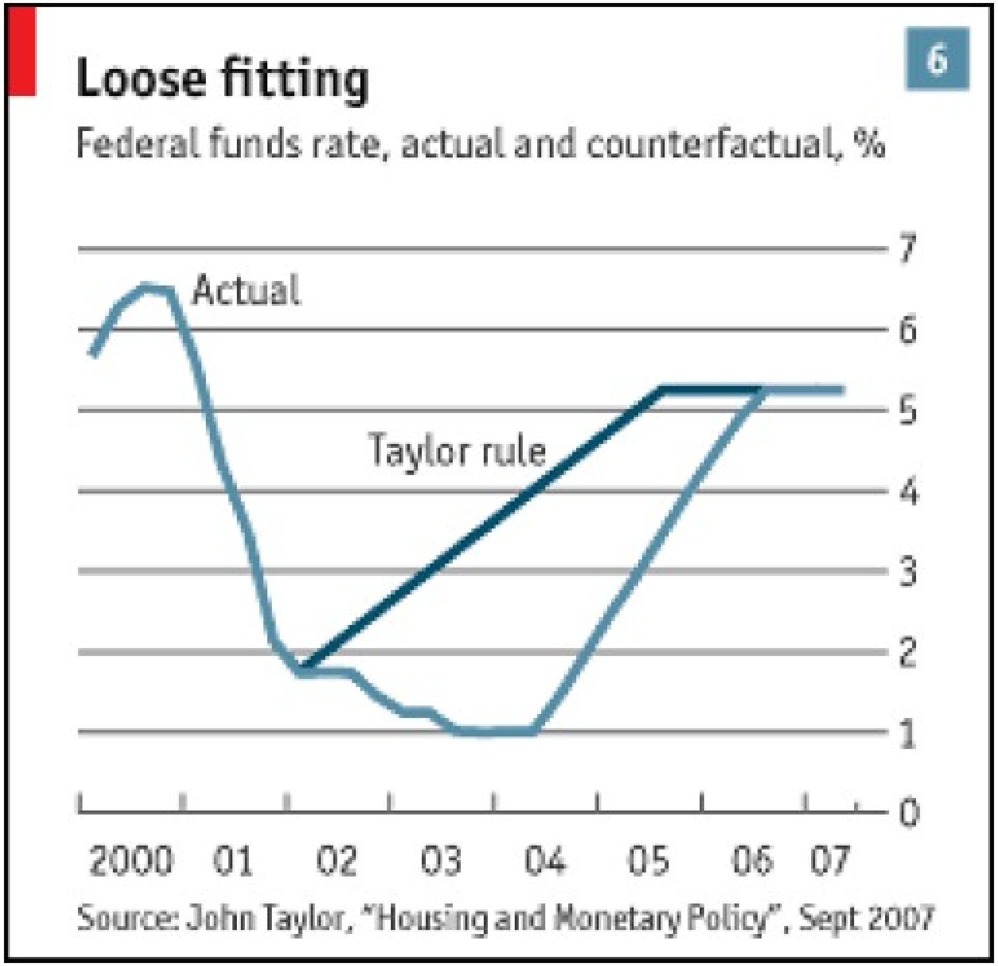
- Summarized in

  THE FINANCIAL CRISIS AND THE POLICY RESPONSES AN EMPIRICAL ANALYSIS OF WHAT WENT WRONG

  NBER Working Paper 14631

  http://www.nber.org/papers/w14631

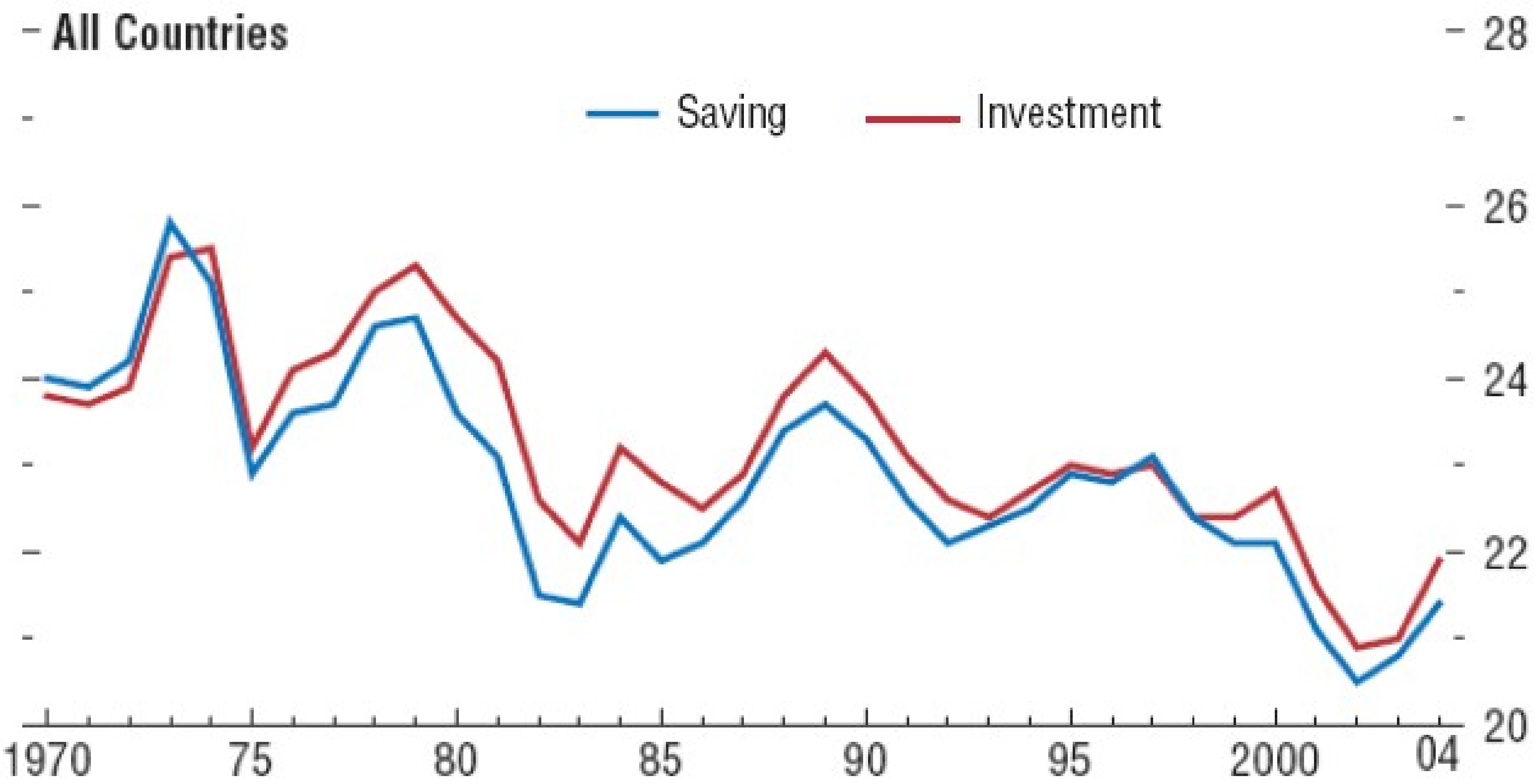
John Taylor opposite view



- Easy monetary policy in 2001-2005
- Actual interest rate (federal funds rate) below what is implied by the Taylor rule

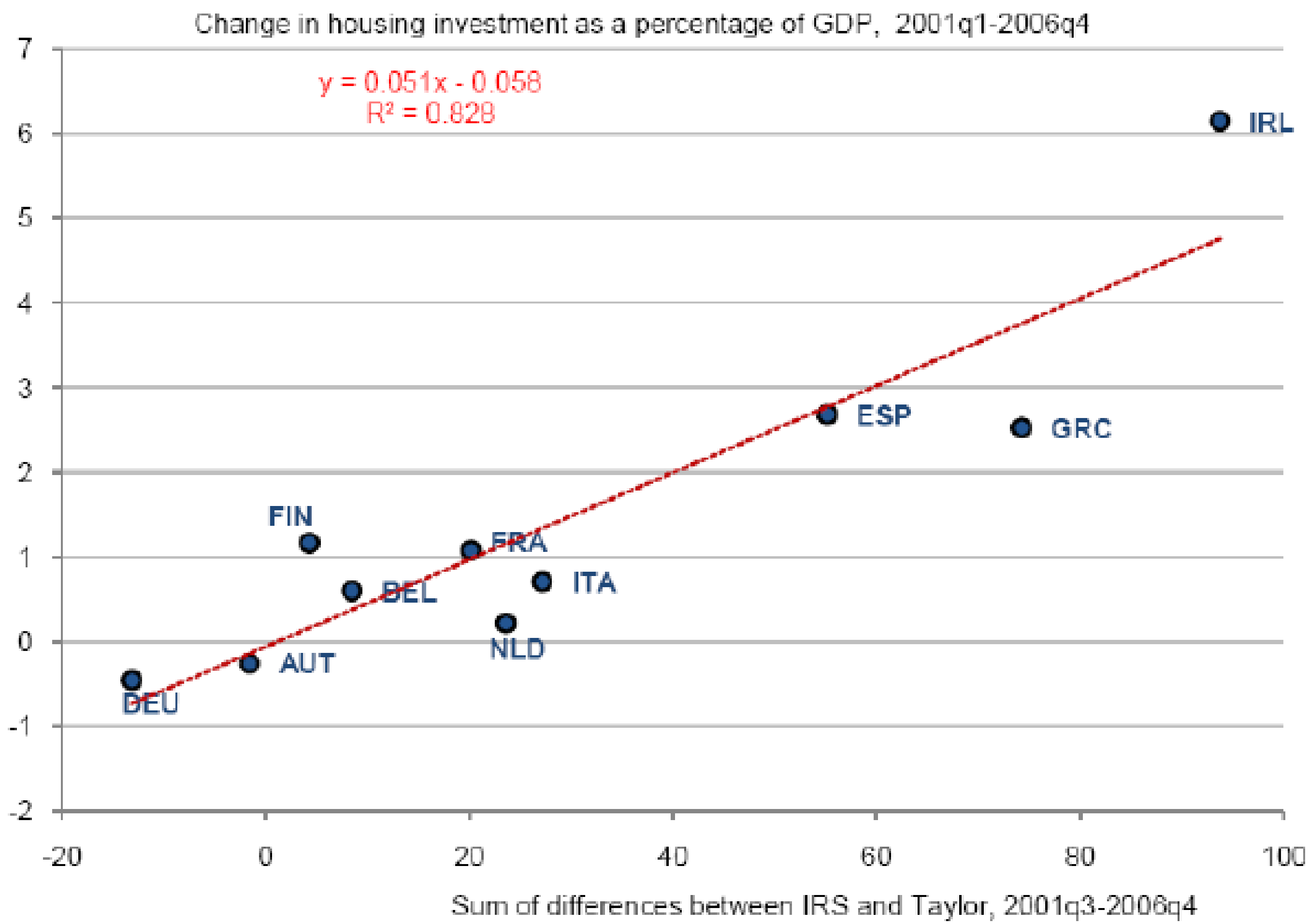
John Taylor view

- No global saving/investment imbalance



Source: *World Economic Outlook*, IMF Sept 2005, Chapter 2, p. 92

John Taylor view



- Monetary policy was too easy in more countries
- The easier the policy was the higher the housing investment

R. Ahrend, B. Cournède and R. Price: "Monetary Policy, Market Excesses and Financial Turmoil", *OECD Economics Department Working Papers*, No. 597, March 2008.

Ben Bernanke's response

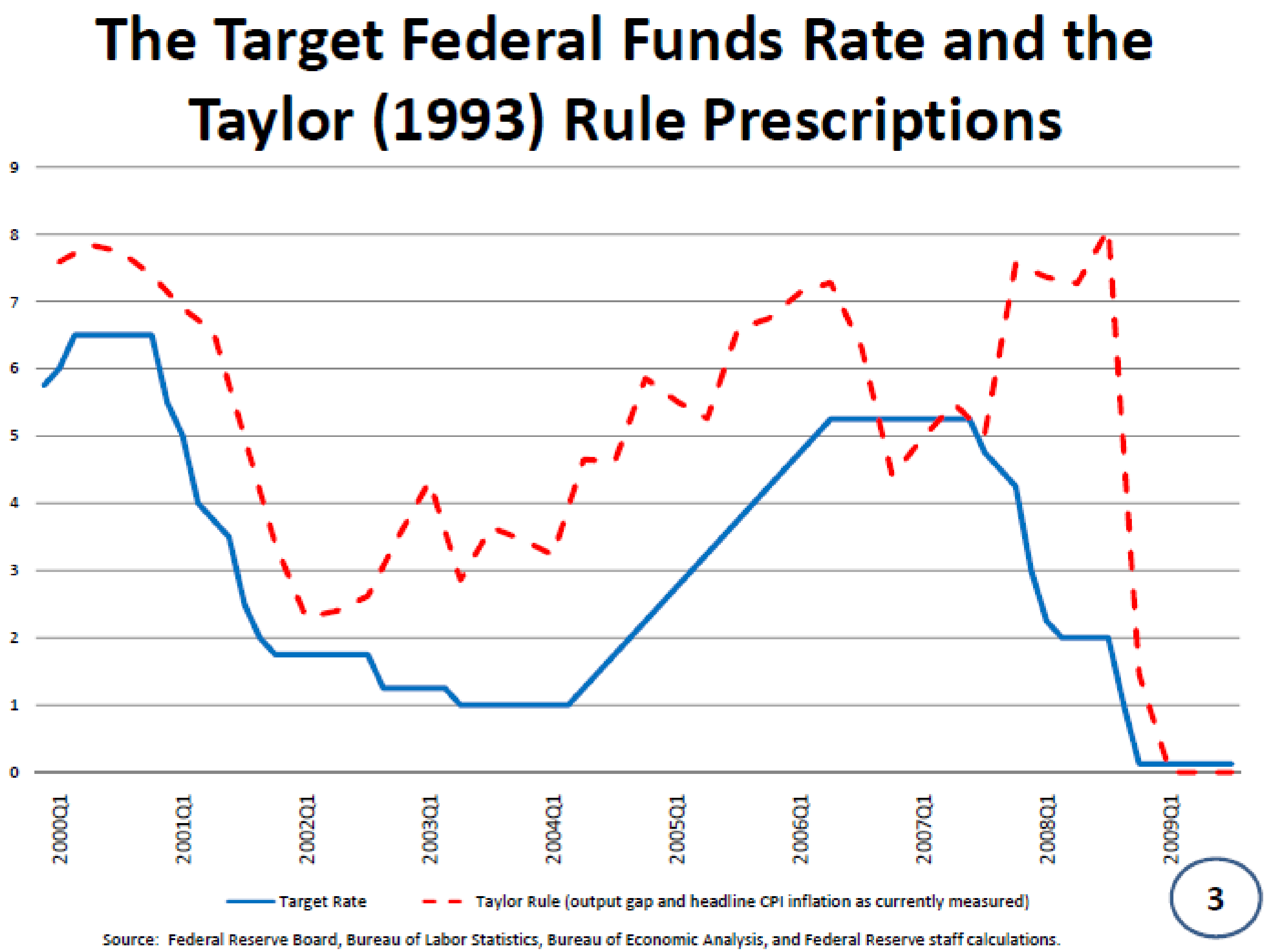
- Summarized in

Monetary Policy and the Housing Bubble

At the Annual Meeting of the American Economic Association, Atlanta, Georgia, January 3, 2010

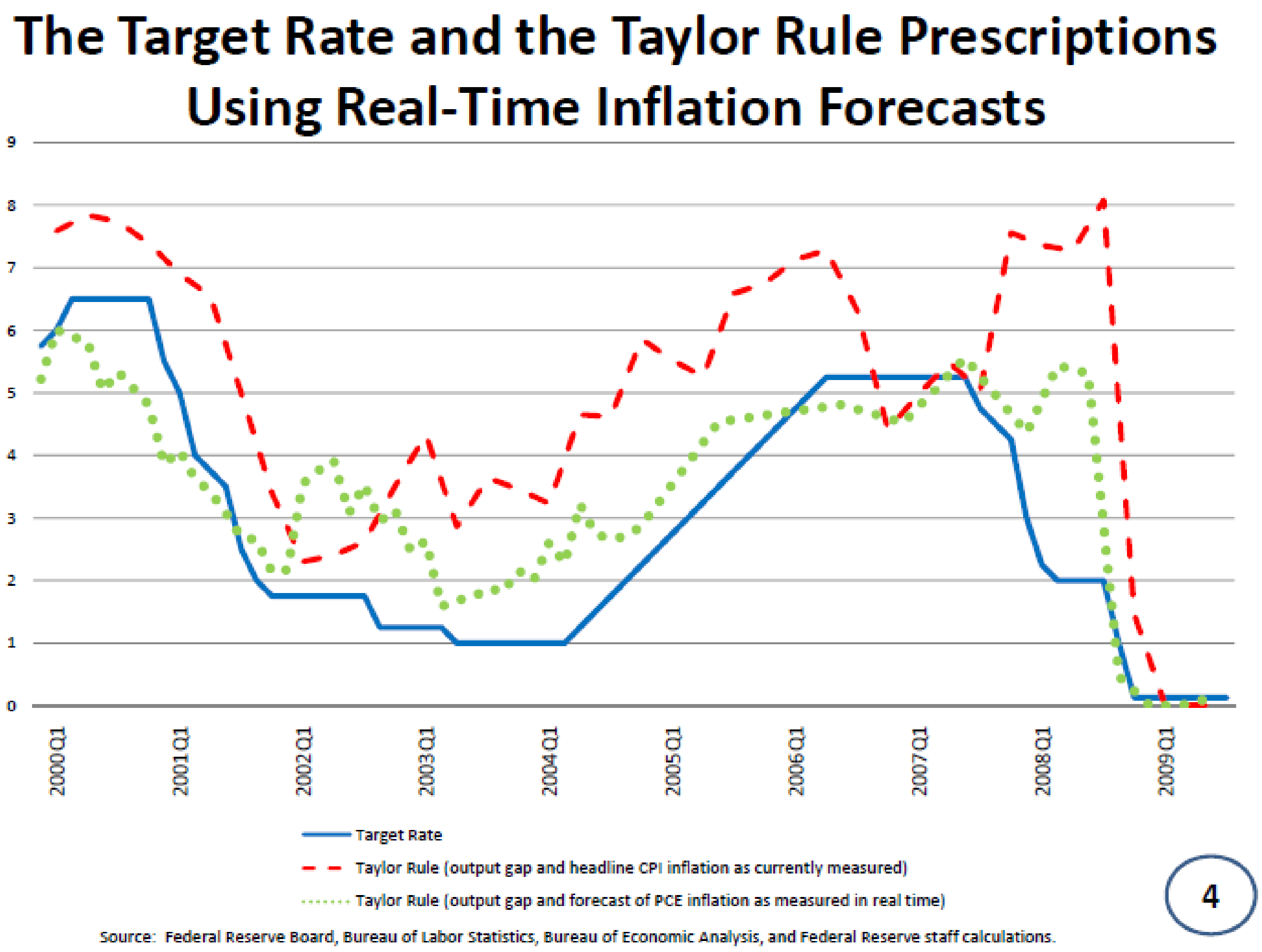
http://www.federalreserve.gov/newsevents/speech/bernanke20100103a.htm

Ben Bernanke's response



Ben Bernanke: „ Monetary Policy and the Housing Bubble ", *At the Annual Meeting of the American Economic Association, Atlanta, Georgia,* January 3, 2010.

Ben Bernanke's response



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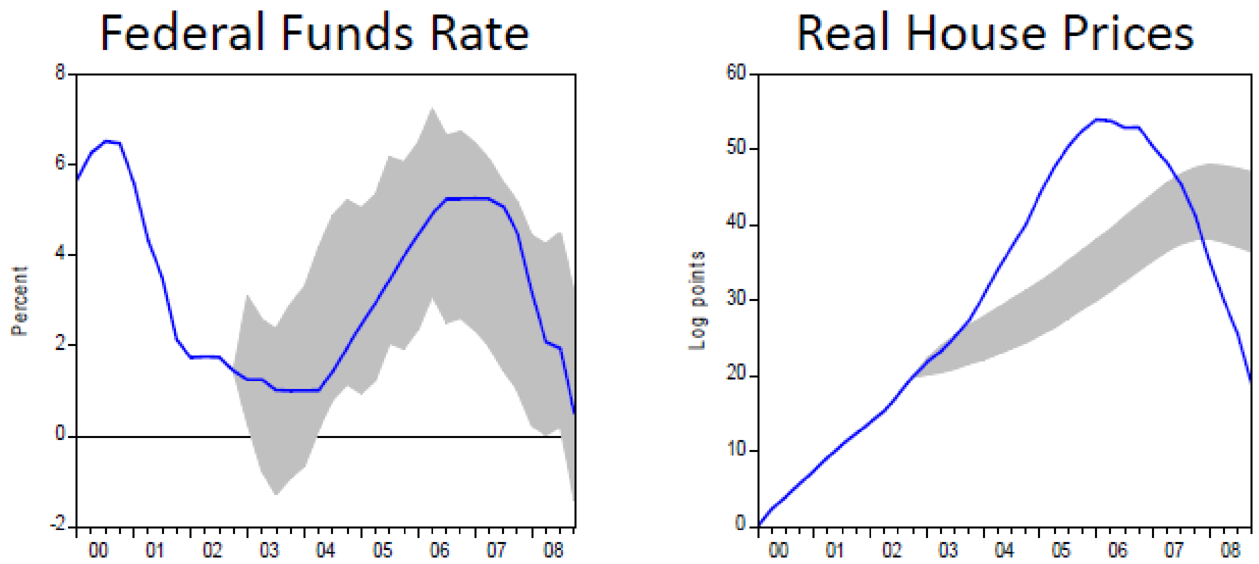
Ben Bernanke's response

Conditional Forecasts for the Federal Funds Rate and House Prices



Note: Shaded areas denote values within 2 standard deviations of the conditional forecast of each variable.

Source: Federal Reserve Board, Bureau of Economic Analysis, FirstAmerican LoanPerformance, and Federal Reserve staff calculations.

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Ben Bernanke's response

Alternative Mortgage Instruments and Associated Initial Monthly Payments

Mortgage Product	Initial Monthly Payment	Payment as a Percentage of FRM Payment
Fixed-rate mortgage (FRM)	$1,079.19	100.0
Adjustable-rate mortgage (ARM)	903.50	83.7
Interest-only/ARM	663.00	61.4
40-year amortization (ARM)	799.98	74.1
Negative amortization ARM	150.00	13.9
Pay-option ARM	<150.00	<13.9

Note: Interest rates used in these calculations were 6.00 percent for FRMs and 4.42 percent for standard ARMs. For purposes of the calculations, we assume a house price of $225,000 and a 20 percent down payment, and that the borrower qualifies for a prime product.

Source: Interest rates for these calculations are from Freddie Mac and are for the period from 2003 through 2006.

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Ben Bernanke's response

Nontraditional Mortgage Features

(Percent of ARM originations)

	Interest Only		Extended Amortization		Negative Amortization	Pay-Option
	Subprime	Alt-A	Subprime	Alt-A	Alt-A	Alt-A
2000	0	3	0	0	---	---
2001	0	8	0	0	---	---
2002	2	37	0	0	---	---
2003	5	48	0	0	19	11
2004	18	51	0	0	40	25
2005	21	48	13	0	46	38
2006	16	51	33	2	55	38

Source: Calculations based on data from First American LoanPerformance.

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Ben Bernanke: „ Monetary Policy and the Housing Bubble ", *At the Annual Meeting of the American Economic Association, Atlanta, Georgia,* January 3, 2010.

Summary

- Aggregate consumption can be constant even if $(1+r^*)\beta \neq 1 \Rightarrow$ more realistic CA prediction than the infinite-horizon model;
- One-off positive/negative shock leads to a temporary CA surplus/deficit;
- Ricardian equivalence does not hold $\Rightarrow$ debt financing leads to CA deficit („twin deficits“);