Introduction to New-Keynesian Economics

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Building Blocks

- Household's problem
- Firms' problem
  - Sticky prices
  - Monopolistic competition
- Policy rule

Before we start: Terminology

- Structural: Each equation has an economic interpretation
- General Equilibrium: Demand=Supply
- Stochastic: There are random shocks
- Rational expectations: Agents use past information and the knowledge about how the economy works (the model) to make inference about future
- Dynamic: We care not only about today but also about yesterday and tomorrow

Household's problem

- Maximize:

Elasticity of substitution between today and future

$$E_t \sum_{i=0}^{\infty} \beta^{t+i} \left[\frac{C_{t+i}^{1-\sigma}}{1-\sigma} + \frac{\gamma}{1-b} \left(\frac{M_{t+i}}{P_{t+i}} \right)^{1-b} - \chi \frac{N_{t+i}^{1+\eta}}{1+\eta} \right]$$

Weight put on leisure in comparison to consumption

- Notation:
- C :consumption
- N :labor, $1-N$:leisure
- M/P :real money balances
- Note that there is no investment or government in the model so $C=Y$

Household's problem

$$E_t \sum_{i=0}^{\infty} \left[\frac{C_{t+i}^{1-\sigma}}{1-\sigma} + \frac{\gamma}{1-b} \left(\frac{M_{t+i}}{P_{t+i}} \right)^{1-b} - \chi \frac{N_{t+i}^{1+\eta}}{1+\eta} \right]$$

- Representative households with preferences defined over consumption, real money balances and leisure.
- Money in the utility:
  - Households are better off when they hold more real money balances
- Expectations are rational

Household's problem

$$C_t = \left[\int_0^1 c_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}}, \theta > 1$$

- C is a basket
- Consisting of differentiated products produced by monopolistically competitive final goods producers.
- There is a continuum of firms and each firm j produces good c_j
- θ is the price elasticity of demand for the individual goods.

Household's problem

- Household's problem can be solved in two stages
  1- Minimize cost for a given C
  2- Given the cost of achieving any given C, choose C, N and M

Household's problem: first stage

- Minimize

$$\min_{c_{jt}} \int_0^1 p_{jt} c_{jt} \, dj$$

subject to

$$\left[\int_0^1 c_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \geq C_t$$

where p_j is the price of good j.

Household's problem: first stage

- The Lagrange function is as follows

$$\Gamma = \int_0^1 p_{jt} c_{jt} dj + \psi_t \left[C_t - \left[\int_0^1 c_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \right]$$

Lagrange multiplier

- And the first order condition for any c_{jt}

$$\partial\Gamma \big/ \partial c_{jt} \equiv p_{jt} - \psi_t \left[\int_0^1 c_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{1}{\theta-1}} c_{jt}^{-\frac{1}{\theta}} = 0$$

Household's problem: first stage

- Using the definition of the composite good

$$C_t = \left[\int_0^1 c_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}}, \theta > 1$$

- We get

$$p_{jt} - \psi_t \left[\int_0^1 c_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{1}{\theta-1}} c_{jt}^{-\frac{1}{\theta}} = 0$$

First order condition (f.o.c.)

$$p_{jt} - \psi_t C_t^{\frac{1}{\theta}} c_{jt}^{-\frac{1}{\theta}} = 0$$

Definition of the composite good substituted in the f.o.c

$$p_{jt}^{-\theta} - \psi_t^{-\theta} C_t^{-1} c_{jt} = 0$$

Household's problem: first stage

- And finally

$$c_{jt} = (p_{jt} / \psi_t)^{-\theta} C_t$$

- We substitute this back in the definition of the composite good:

$$C_t = \left[\int_0^1 \left[\left(\frac{p_{jt}}{\psi_t} \right)^{-\theta} C_t \right]^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} = \left(\frac{1}{\psi_t} \right)^{-\theta} \left[\int_0^1 p_{jt}^{1-\theta} dj \right]^{\frac{\theta}{\theta-1}} C_t$$

Household's problem: first stage

- And solve for ψ_t :

$$\psi_t = \left[\int_0^1 p_{jt}^{1-\theta} dj \right]^{\frac{1}{\theta-1}} = P_t$$

- The Lagrange multiplier is the appropriate price index (shadow price) for consumption.
- The demand for j-th good is:

$$c_{jt} = \left(\frac{p_{jt}}{P_t} \right)^{-\theta} C_t$$

Household's problem: first stage

$$c_{jt} = \left(\frac{p_{jt}}{P_t} \right)^{-\theta} C_t$$

- This is the individual "demand curve"
- Demand for a particular good j depends on its price relative to composite good price index
- Price elasticity of demand for good j is also important. As $\theta \to \infty$, individual goods become closer substitutes and firms have less market power

Household's problem: second stage

$$\max_{C,M,(1-N)} E_t \sum_{i=0}^{\infty} \beta^{t+i} \left[\frac{C_{t+i}^{1-\sigma}}{1-\sigma} + \frac{\gamma}{1-b} \left(\frac{M_{t+i}}{P_{t+i}} \right)^{1-b} - \chi \frac{N_{t+i}^{1+\eta}}{1+\eta} \right]$$

Budget constraint

$$\text{s.t. } C_t + \frac{M_t}{P_t} + \frac{B_t}{P_t} = \left(\frac{W_t}{P_t} \right) N_t + \frac{M_{t-1}}{P_t} + (1+i_{t-1}) \left(\frac{B_{t-1}}{P_t} \right) + \Pi_t$$

- B_t is household's nominal holdings of one-period bonds
- Bonds pay a nominal interest rate of i_t
- Real profits received from firms are Π_t
- W_t nominal wages

Household's problem: second stage

- Choose consumption, labor supply, money, and bond holdings. First order conditions are:

$$C_t^{-\sigma} = \beta(1+i_t)E_t\left(\frac{P_t}{P_{t+1}}\right)C_{t+1}^{-\sigma}$$

$$\frac{\gamma\left(\frac{M_t}{P_t}\right)^{-b}}{C_t^{-\sigma}} = \left(\frac{i_t}{1+i_t}\right)$$

$$\frac{\chi N_t^{\eta}}{C_t^{-\sigma}} = \left(\frac{W_t}{P_t}\right)$$

Household's problem: second stage

$$C_t^{-\sigma} = \beta(1+i_t)E_t\left(\frac{P_t}{P_{t+1}}\right)C_{t+1}^{-\sigma}$$

- Euler equation (inter-temporal) for the optimal intertemporal allocation of consumption:
  - On the LHS: marginal utility of 1 unit of consumption today
  - On the RHS: Expected marginal utility of consumption tomorrow if decide to save that 1 unit of consumption today
  - In equilibrium these two need to be equal

Household's problem: second stage

$$\frac{\gamma\left(\frac{M_t}{P_t}\right)^{-b}}{C_t^{-\sigma}} = \left(\frac{i_t}{1+i_t}\right)$$

- Intratemporal optimality condition to pin down money holdings
  - On the LHS: marginal rate of substitution between money and consumption
  - On the RHS: opportunity cost of holding money

Household's problem: second stage

$$\frac{\chi N_t^{\eta}}{C_t^{-\sigma}} = \left(\frac{W_t}{P_t}\right)$$

- Intratemporal optimality condition to determine labor supply
  - On the LHS: Marginal rate of substitution between leisure and consumption
  - On the RHS: Real wage

Firms' problem

- Maximize profits subject to:
  - 1- Constant returns to scale technology

$$c_{jt} = Z_t N_{jt}, E(Z_t) = 1$$

    - Where Z is an aggregate productivity shock
  - 2- Demand curve of the HH
  - 3- Sticky prices a la Calvo (1983): explained in next slide…

- Firms are identical except that they might have set their prices at different dates in the past.

Firms' problem

- Calvo pricing
- Each period, firms that adjust their price are randomly selected
  - A fraction $1-\omega$ of firms adjust while remaining ω do not.
  - ω measures the degree of nominal rigidity
  - Higher ω $\Rightarrow$ fewer firms adjust each period, expected time between price adjustments is longer
  - Firms that adjust do so to maximize expected discounted value of current and future profits

Sticky prices

- Why should prices adjust slowly?
- One common explanation is "menu costs": small costs that must be paid in order to adjust nominal prices.
  - The costs of making a new catalog, price list, or menu.

Sticky prices

- There are also externalities that go along with changing prices:
  - A firm that lowers its prices because of a decrease in the money supply will be raising the real income of the customers of that product.
  - This will allow the buyers to purchase more, which will not necessarily be from the firm that lowered their prices.
  - As firms do not receive the full benefit from reducing their prices their incentive to adjust prices in response to macroeconomic events is reduced.

Monopolistic competition

- Without some monopoly power it doesn’t make sense to assume sticky prices!
  - Under perfect competition, any firm with a price slightly higher than the others would be unable to sell anything. Any firm with a price slightly lower than the others would be obliged to sell much more than they can profitably produce.
- Firms use their market power to maintain their prices above marginal cost, so that even if they fail to set prices optimally they will remain profitable.

Firms' problem: First stage

- Cost minimization: minimize wage bill subject to producing a given amount

$$\min_{N_t} \left(\frac{W_t}{P_t} \right) N_t + \varphi_t \left(c_{jt} - Z_t N_{jt} \right)$$

- φ_t is the firms' marginal cost.
- First order condition is:

$$\varphi_t = \frac{W_t / P_t}{Z_t}$$

Firms' problem: Second stage

- Price setting: choose price to maximize present discounted value of profits

Cost of production in real terms

$$\max_{p_{jt}} \Pi_t = E_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+i} \left[\left(\frac{p_{jt}}{P_{t+i}} \right) c_{jt+i} - \varphi_{t+i} c_{jt+i} \right]$$

Probability of not-being able to change the price

Revenues in real terms

- Where $\Delta_{i,t+i} = \beta^i \left(C_{t+i} / C_t \right)^{-\sigma}$ is the discount factor
- It is the same as households' discount factor since households are assumed to own the firms

Firms' problem: Second stage

- Substituting households' demand curve for the j-th firm production in the j-th firm's profit maximization:

$$\max_{p_{jt}} E_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+i} \left[\left(\frac{p_{jt}}{P_{t+i}} \right)^{1-\theta} - \varphi_{t+i} \left(\frac{p_{jt}}{P_{t+i}} \right)^{-\theta} \right] C_{t+i}$$

- Let p_t^* be the optimal price chosen by firms adjusting at time t. Then the firm's f.o.c. is:

$$\partial \Pi_t \Big/ \partial p_t^* \equiv E_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+i} \left[(1-\theta) \left(\frac{p_t^*}{P_{t+i}} \right) + \theta \varphi_{t+i} \right] \left(\frac{1}{p_t^*} \right) \left(\frac{p_t^*}{P_{t+i}} \right)^{-\theta} C_{t+i} = 0$$

Firms' problem: Second stage

- The f.o.c. is solved for p_t^* :

$$p_t^* = \left(\frac{\theta}{\theta - 1}\right) \frac{E_t \sum_{i=0}^{\infty} \omega^i \Delta_{t,t+i} \varphi_{t+i} P_{t+i}^{\theta} C_{t+i}}{E_t \sum_{i=0}^{\infty} \omega^i \Delta_{t,t+i} P_{t+i}^{\theta-1} C_{t+i}}$$

- Using $\Delta_{i,t+i} = \beta^i \left(C_{t+i} / C_t\right)^{-\sigma}$ it can be rearranged in real terms as:

$$\left(\frac{p_t^*}{P_t}\right) = \left(\frac{\theta}{\theta - 1}\right) \frac{E_t \sum_{i=0}^{\infty} \omega^i \beta^i C_{t+i}^{1-\sigma} \varphi_{t+i} \left(\frac{P_{t+i}}{P_t}\right)^{\theta}}{E_t \sum_{i=0}^{\infty} \omega^i \beta^i C_{t+i}^{1-\sigma} \left(\frac{P_{t+i}}{P_t}\right)^{\theta-1}}$$

Firms' problem: Second stage

- If all firms could adjust every period, i.e. under flexible prices: $(\omega = 0)$

$$\left(\frac{p_t^*}{P_t}\right) = \left(\frac{\theta}{\theta - 1}\right)\varphi_t = \mu\varphi_t$$

  - Each firm sets its price to a markup μ over its nominal marginal cost $P_t\varphi_t$
  - A standard result in a model of monopolistic competition
  - As price>marginal cost, output is inefficiently low even under flexible prices!
- Further it holds that under flexible prices all firms set the same price, $p_t^* = P_t$ and $\varphi_t = 1/\mu$

Firms' problem: Second stage

- Remember:

$$\psi_t = \left[\int_0^1 p_{jt}^{1-\theta} dj \right]^{\frac{1}{\theta-1}} = P_t$$

- Since the adjusting firms are chosen randomly form a continuum of firms following aggregation holds:

$$P_t^{1-\theta} = (1-\omega)\left(p_t^*\right)^{1-\theta} + \omega P_{t-1}^{1-\theta}$$

- That is in fact what is so attractive on Calvo pricing. Thanks Calvo!

Sum up …

- A non-linear version of the model looks as follows:

Households

$$c_{jt} = \left(\frac{p_{jt}}{P_t} \right)^{-\theta} C_t$$

$$C_t^{-\sigma} = \beta (1 + i_t) E_t \left(\frac{P_t}{P_{t+1}} \right) C_{t+1}^{-\sigma}$$

$$\frac{\chi N_t^{\eta}}{C_t^{-\sigma}} = \left(\frac{W_t}{P_t} \right)$$

$$\frac{\gamma \left(\frac{M_t}{P_t} \right)^{-b}}{C_t^{-\sigma}} = \left(\frac{i_t}{1 + i_t} \right)$$

Firms

$$c_{jt} = Z_t N_{jt}$$

$$\left(\frac{p_t^*}{P_t} \right) = \left(\frac{\theta}{\theta - 1} \right) \frac{E_t \sum_{i=0}^{\infty} \omega^i \beta^i C_{t+i}^{1-\sigma} \varphi_{t+i} \left(\frac{P_{t+i}}{P_t} \right)^{\theta}}{E_t \sum_{i=0}^{\infty} \omega^i \beta^i C_{t+i}^{1-\sigma} \left(\frac{P_{t+i}}{P_t} \right)^{\theta - 1}}$$

$$P_t^{1-\theta} = (1 - \omega) \left(p_t^* \right)^{1-\theta} + \omega P_{t-1}^{1-\theta}$$

Sum up …

- Equations above represent a system of non-linear forward-looking equations
- It is impossible to handle them in that form
- To find a solution, i.e. a system of equations where variables do not depend on future values, equations above have to be approximated
- The most common approximation is a first order Taylor approximation, i.e. a linear approximation around a fixed point
- We know how to solve a system of linear forward-looking difference equations

Log-linear approximation of household's f.o.c.

- Approximation of the Euler equation around the zero steady state:

$$\hat{c}_t = E_t \hat{c}_{t+1} - \left(\frac{1}{\sigma}\right)\left(\hat{i}_t - E_t \pi_{t+1}\right)$$

- And because it holds: $\hat{c}_t = \hat{y}_t$ (no investment or government)
- It can be written also as:

$$\hat{y}_t = E_t \hat{y}_{t+1} - \left(\frac{1}{\sigma}\right)\left(\hat{i}_t - E_t \pi_{t+1}\right)$$

Log-linear approximation of household's f.o.c.

- Equation above can be also expressed in terms of the output gap $x_t = \hat{y}_t - \hat{y}_t^f$

Flexible-price equilibrium output

$$x_t = E_t x_{t+1} - \left(\frac{1}{\sigma}\right)\left(\hat{i}_t - E_t \pi_{t+1} - \sigma \hat{r}_t^n\right)$$

- Where $\hat{r}_t^n$ is the policy neutral real rate driven by the expected evolution of the flexible-price equilibrium output

$$\hat{r}_t^n = E_t \hat{y}_{t+1}^f - \hat{y}_t^f$$

- We get an IS curve!
- Output gap depends on the real interest rate gap

Flexible-price equilibrium output

- Equation for the flexible-price equilibrium output can be derived analytically using the fact that under flexible-prices $\varphi_t = 1/\mu$
- While from production function $\varphi_t = \dfrac{W_t / P}{Z_t}$
- And the real wage must be equal to the households' MRS between the consumption and leisure
  $\chi N_t^{\eta} \big/ C_t^{-\sigma} = W_t / P_t$
- Combining these and using the fact that under flexible prices $Y_t^f \equiv Y_t = C_t$

$$Y_t^f = \left(\frac{1}{\chi\mu} \right)^{1/(\sigma+\eta)} Z_t^{(1+\eta)/(\sigma+\eta)} \quad \text{and} \quad \hat{y}_t^f = \frac{(1+\eta)}{(\sigma+\eta)} \hat{z}_t$$

Log-linear approximation of household's f.o.c.

- Approximation of the intratemporal (labor market) first order condition:

$$\eta\hat{n}_t + \sigma\hat{c}_t = \hat{w}_t - \hat{p}_t$$

- Real wage is equal to the MRS between leisure and consumption
- Labor market is always cleared in this version!
- Approximating the third first order condition:

$$\hat{m}_t - \hat{p}_t = \left(\frac{1}{bi^{ss}}\right)\left(\sigma\hat{c}_t - \hat{i}_t\right)$$

- We get a money demand equation!

The role of money

- For a given interest rate, consumption and price level one can calculate the implied money using the equation above.
- But money does not appear in IS curve or in the Phillips curve!
- This is because money and consumption are separable in the utility function
- Money is needed for transactions but is not a driving force for the model

Log-linear approximation of firms' price setting

- Using the pricing function of firms and the price aggregation

$$\left(\frac{p_t^*}{P_t}\right)=\left(\frac{\theta}{\theta-1}\right)\frac{E_t\sum_{i=0}^{\infty}\omega^i\beta^iC_{t+i}^{1-\sigma}\varphi_{t+i}\left(\frac{P_{t+i}}{P_t}\right)^{\theta}}{E_t\sum_{i=0}^{\infty}\omega^i\beta^iC_{t+i}^{1-\sigma}\left(\frac{P_{t+i}}{P_t}\right)^{\theta-1}}$$

$$P_t^{1-\theta}=(1-\omega)\left(p_t^*\right)^{1-\theta}+\omega P_{t-1}^{1-\theta}$$

- And quite terrible math …
- One gets the so called New Keynesian Phillips curve

New Keynesian Phillips curve

$$\pi_t = \beta E_t \pi_{t+1} + \tilde{\kappa}\hat{\varphi}_t \qquad \tilde{\kappa} = \frac{(1-\omega)(1-\beta\omega)}{\omega}$$

- Forward looking inflation process
  - When a firm sets its price it must consider future inflation because it may not be able to re-set its price for a while
- Real marginal cost is an important variable driving the inflation process
- The weight on inflation expectations vs. real marginal cost depends on the degree of price stickiness $\uparrow\omega \Rightarrow \tilde{\kappa}\uparrow$

Marginal costs and output gap

- The IS curve is written in terms of the output gap, while inflation is driven by marginal costs
- Marginal costs can be approximated by the output gap if certain condition is met (see Appendix)
  - Analytically - MRS between leisure and consumption must be equal to the real wage at any time (labor market must clear)
  - Practically - real wages must be pro-cyclical (at least)
  - Then …

$$\pi_t = \beta E_t \pi_{t+1} + \tilde{\kappa}\hat{\varphi}_t \qquad \tilde{\kappa} = \frac{(1-\omega)(1-\beta\omega)}{\omega}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t \qquad \kappa = (\eta + \sigma)\tilde{\kappa}$$

Policy rule

- To close the model we need an interest rate rule.
  - But not just ANY rule!
  - It needs to satisfy certain conditions to avoid unstable dynamics or multiple equilibrium
- Example of an unstable rule:

$$\hat{i}_t = \rho_r \hat{i}_{t-1} + \nu_t$$

- Remember our other two equations:

$$x_t = E_t x_{t+1} - \left(\frac{1}{\sigma}\right)\left(\hat{i}_t - E_t \pi_{t+1} - \sigma r_t^n\right)$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t$$

Policy rule

- Write these three equations in matrix form :

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \sigma^{-1} \\ 0 & 0 & \beta \end{bmatrix} \begin{bmatrix} \hat{i}_t \\ E_t x_{t+1} \\ E_t \pi_{t+1} \end{bmatrix} = \begin{bmatrix} \rho_r & 0 & 0 \\ \sigma^{-1} & 1 & 0 \\ 0 & -\kappa & 1 \end{bmatrix} \begin{bmatrix} \hat{i}_{t-1} \\ x_t \\ \pi_t \end{bmatrix} + \begin{bmatrix} \nu_t \\ -u_t \\ 0 \end{bmatrix}$$

- Mathematically, these matrices need to satisfy certain properties to guarantee a unique solution.

Policy rule

- Will not get into technical details but there are multiple solutions to the above system!
  - Sunspot equilibria are possible
    - To see this think about what would happen if inflation expectations were to rise
    - Since the rule does not have inflation on the RHS, the real interest rate will fall
    - A decline in real interest rate leads to an increase in output gap
    - An increase in output gap increases actual inflation
    - Self-fulfilling high inflation!
- In general exogenous policy rules have this problem. Better to use rules that depend on endogenous variables (output gap and inflation)

Policy rule

- Setting a rule that would raise the nominal interest rate enough such that real interest would increase would be enough to solve the multiple equilibria problem! For example:

$$\hat{i}_t = \delta \pi_t + v_t$$

- A unique equilibrium exists as long as $\delta > 1$
- This is called the "Taylor principle"
- Taylor was the first to emphasize that the nominal interest rates should increase more than one-to-one in response to inflation

Policy rule

- The most common rule: “Taylor rule”

$$\hat{i}_t = \delta_\pi \pi_t + \delta_x x_t + v_t$$

- Taylor rule was proposed by Taylor in an empirical context.
  - When looked at how the US Fed set the interest rates historically it looked like they were following a rule that looked very much like what Taylor proposed
- Has been shown to provide a reasonable empirical description of the many Central Banks’ behavior

Policy rule

- An extension is “Forward looking Taylor rule”

$$\hat{i}_t = \delta_\pi E_t \pi_{t+1} + \delta_x x_t + \nu_t$$

- Policymaker responds to expected inflation as opposed to contemporaneous inflation.
  - It nests the Taylor rule as a special case
- With this type of rule and given the rest of the model, the condition to ensure unique equilibrium is:

$$\kappa(\delta_\pi - 1) + (1-\beta)\delta_x > 0$$

Model I …

- A log-linear version of the model in its simplest form has following equations:

$$x_t = E_t x_{t+1} - \left(\frac{1}{\sigma}\right)\left(\hat{i}_t - E_t \pi_{t+1} - \sigma r_t^n\right)$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t$$

$$\hat{i}_t = \delta_\pi \pi_t + \delta_x x_t + v_t$$

$$x_t = \hat{y}_t - \hat{y}_t^f$$

$$\hat{y}_t^f = \frac{(1+\eta)}{(\sigma+\eta)} \hat{z}_t$$

$$\hat{r}_t^n = E_t \hat{y}_{t+1}^f - \hat{y}_t$$

$$\hat{z}_t = \rho \hat{z}_{t-1} + \varepsilon_t$$

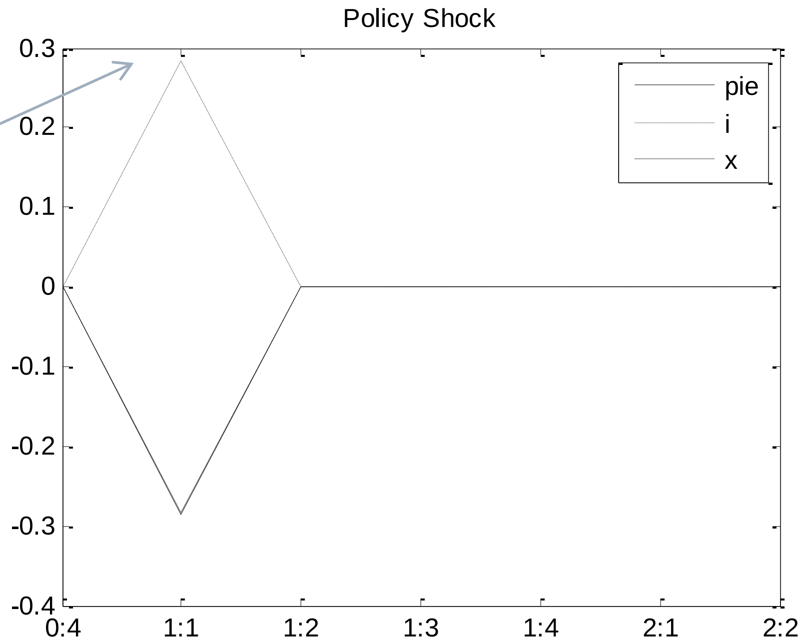
Technology shocks are assumed to be persistent

- Labor market condition and production function can be omitted in this version
- Labor market is always cleared as long as wages are flexible

Model I …

- But the empirical properties of the model are not satisfactory
- Clearly, more persistency is needed

Impulse responses to a one period interest rate shock



Extension – Inflation persistence

- Christiano, Eichenbaum and Evans (2005)
- Extension of Calvo pricing
  - Each period, firms that adjust their price are randomly selected
    - With probability $1-\omega$ firm can adjust price
    - With probability ω it indexes based on past inflation

$$p_{jt} = \pi_{t-1} p_{jt-1}$$

- Aggregate price index becomes

$$P_t^{1-\theta} = (1-\omega)\left(p_t^*\right)^{1-\theta} + \omega \pi_{t-1} P_{t-1}^{1-\theta}$$

Extension – Inflation persistence

- And the Phillips curve looks as:

$$\pi_t = \left(\frac{1}{1+\beta}\right)\pi_{t-1} + \left(\frac{\beta}{1+\beta}\right)E_t\pi_{t+1} + \frac{(1-\beta\omega)(1+\omega)}{(1+\beta)\omega}\hat{\varphi}_t$$

- Or

$$\pi_t = \left(\frac{1}{1+\beta}\right)\pi_{t-1} + \left(\frac{\beta}{1+\beta}\right)E_t\pi_{t+1} + (\eta+\sigma)\left[\frac{(1-\beta\omega)(1+\omega)}{(1+\beta)\omega}\right]x_t$$

- When approximating the real marginal cost by the output gap

Model II …

- A log-linear version of the extended model is similar to the Model I, except of a more complicated Phillips curve:

$$x_t = E_t x_{t+1} - \left(\frac{1}{\sigma}\right)\left(\hat{i}_t - E_t \pi_{t+1} - \sigma r_t^n\right)$$

$$\pi_t = \left(\frac{1}{1+\beta}\right)\pi_{t-1} + \left(\frac{\beta}{1+\beta}\right)E_t \pi_{t+1} + (\eta + \sigma)\left[\frac{(1-\beta\omega)(1+\omega)}{(1+\beta)\omega}\right]x_t$$

$$\hat{i}_t = \delta_\pi \pi_t + \delta_x x_t + \nu_t$$

$$x_t = \hat{y}_t - \hat{y}_t^f$$

$$\hat{y}_t^f = \frac{(1+\eta)}{(\sigma+\eta)}\hat{z}_t$$

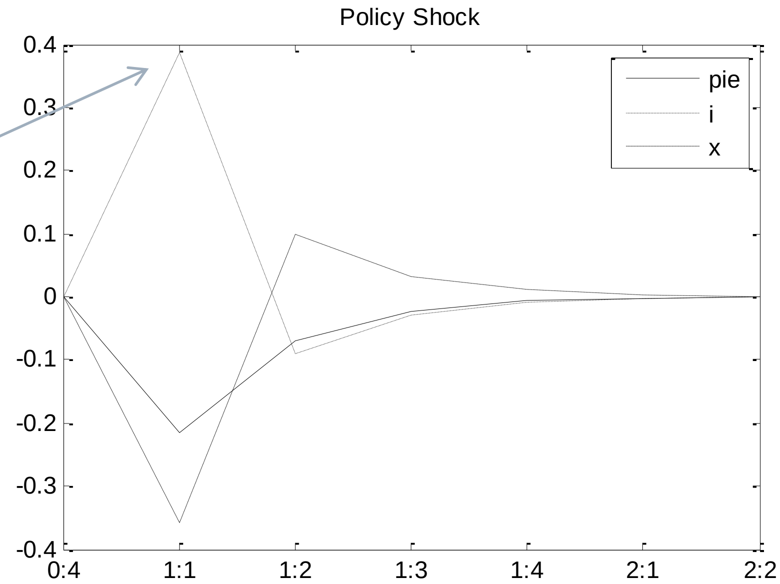
$$\hat{r}_t^n = E_t \hat{y}_{t+1}^f - \hat{y}$$

$$\hat{z}_t = \rho \hat{z}_{t-1} + \varepsilon_t$$

Model II …

- From the macro perspective model properties improve

Impulse response to a one period interest rate shock



- Micro foundation of pricing behavior is questionable
  - Micro studies seems to confirm that firms either change the price or not, but they do not seem to index

Appendix: Approximation of marginal costs by the output gap

$$\varphi_t = \frac{W_t / P_t}{Z_t}$$

Substitute $\eta \hat{n}_t + \sigma \hat{c}_t$ *for* $\hat{w}_t - \hat{p}_t$

$$\hat{\varphi}_t = \hat{w}_t - \hat{p}_t - \hat{z}_t$$

$$\hat{\varphi}_t = \eta \hat{n}_t + \sigma \hat{c}_t - \hat{z}_t$$

Substitute $\hat{y}_t - \hat{z}_t$ *for* $\hat{n}_t$ *and* $\hat{y}_t$ *for* $\hat{c}_t$

$$\hat{\varphi}_t = \eta \left(\hat{y}_t - \hat{z}_t \right) + \sigma \hat{y}_t - \hat{z}_t$$

$$\hat{\varphi}_t = \left(\eta + \sigma \right) \hat{y}_t - \left(1 + \eta \right) \hat{z}_t$$

Flexible-price equilibrium output $\hat{y}_t^f$

$$\hat{\varphi}_t = \left(\eta + \sigma \right) \left[\hat{y}_t - \frac{\left(1 + \eta \right)}{\left(\eta + \sigma \right)} \hat{z}_t \right]$$

$$\hat{\varphi}_t = \left(\eta + \sigma \right) x_t$$