L4. Neoclassical Growth Model in an Open Economy

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Closed Economy

- Economy and people (families) exist forever
- Exogenous growth rate of population *n*
- Exogenous growth rate of technology *g*
- No depreciation of capital.

$$Max\,U = L_0 \sum_{t=0}^{\infty} \beta^t (1+n)^t u(C_t); \qquad \beta = \frac{1}{1+\rho} \qquad (1)$$

$$Y_t = F(K_t, E_t L_t) \qquad (2)$$

$$K_{t+1} - K_t = F(K_t, E_t L_t) - C_t \qquad (3)$$

Closed Ec.: Intensive Form

$$\frac{K_{t+1}}{E_t L_t} - \frac{K_t}{E_t L_t} = \frac{F(K_t, E_t L_t)}{E_t L_t} - \frac{C_t}{E_t L_t} \qquad (4)$$

$$\frac{K_{t+1}}{E_{t+1} L_{t+1}} \frac{E_{t+1} L_{t+1}}{E_t L_t} - \frac{K_t}{E_t L_t} = \frac{F(K_t, E_t L_t)}{E_t L_t} - \frac{C_t}{E_t L_t} \qquad (5)$$

$$k_{t+1}(1+n)(1+g) - k_t = f(k_t) - c_t \qquad (6)$$

$$k_{t+1}(1+z) - k_t = f(k_t) - c_t; \qquad (1+z) \equiv (1+n)(1+g) \qquad (7)$$

$$k_{t+1} - k_t = \frac{f(k_t) - c_t}{1+z} - \frac{z}{1+z} k_t \qquad (8)$$

- Investment must cover the population and technology growth.

Closed Economy: Household Choice

$$u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma} \tag{9}$$

$$\frac{c_{t+1}}{c_t} = \frac{\beta^{\frac{1}{\sigma}}\left[1 + f_k'(k_{t+1})\right]^{\frac{1}{\sigma}}}{1+g} \tag{10}$$

$$k_{t+1} - k_t = \frac{f(k_t) - c_t}{1+z} - \frac{z}{1+z}k_t \tag{11}$$

- Consumption growth governed by the Euler equation (here in the intensive form).

Closed Ec.: Equilibrium

$$\frac{c_{t+1}}{c_t} = 1 \qquad (12)$$

$$k_{t+1} - k_t = 0 \qquad (13)$$

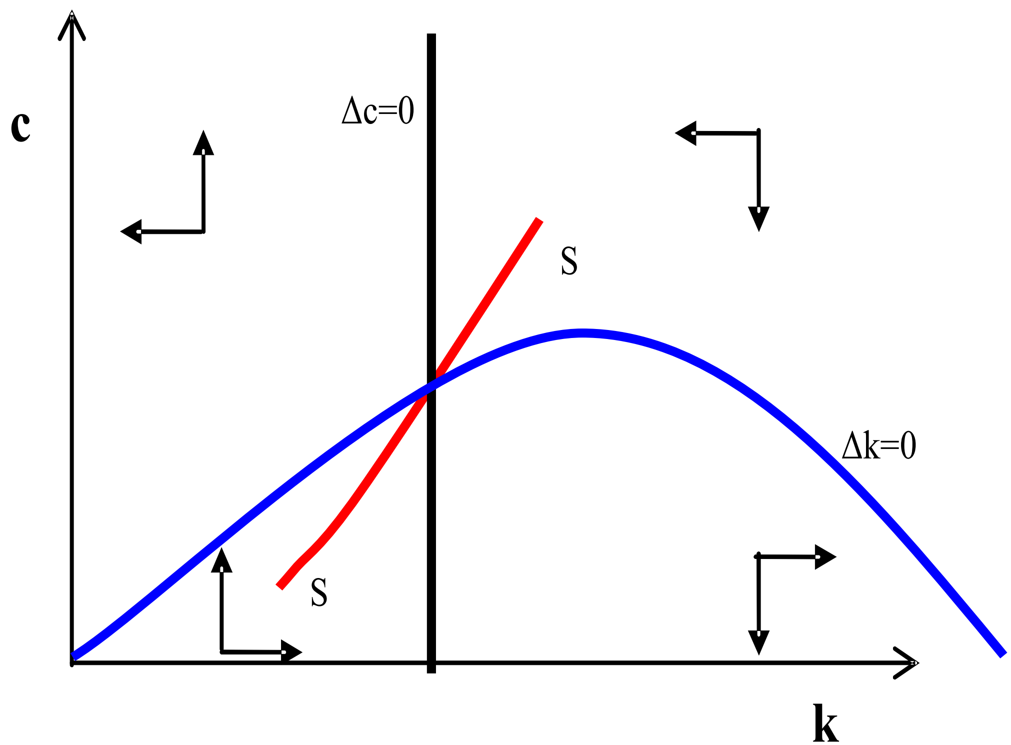
$$\beta^{\frac{1}{\sigma}} \left[1 + f_k'\left(k_{t+1}\right)\right]^{\frac{1}{\sigma}} = 1 + g \qquad (14)$$

$$1 + f_k'\left(k_{t+1}\right) = \frac{\left(1 + g\right)^{\sigma}}{\beta} \qquad (15)$$

$$1 + f_k'\left(k_{t+1}\right) = \left(1 + g\right)^{\sigma}\left(1 + \rho\right) \qquad (16)$$

- Equilibrium real IR depends on technology growth and impatience of people.

Phase Diagram



- Gradual convergence to the steady state along the saddle path (SS);
- Convergence speed depends on model parameters (with α=1/3, it is around
  4 % a year);
- Empirical speed of convergence: 1.5-3.0%.

Open Economy

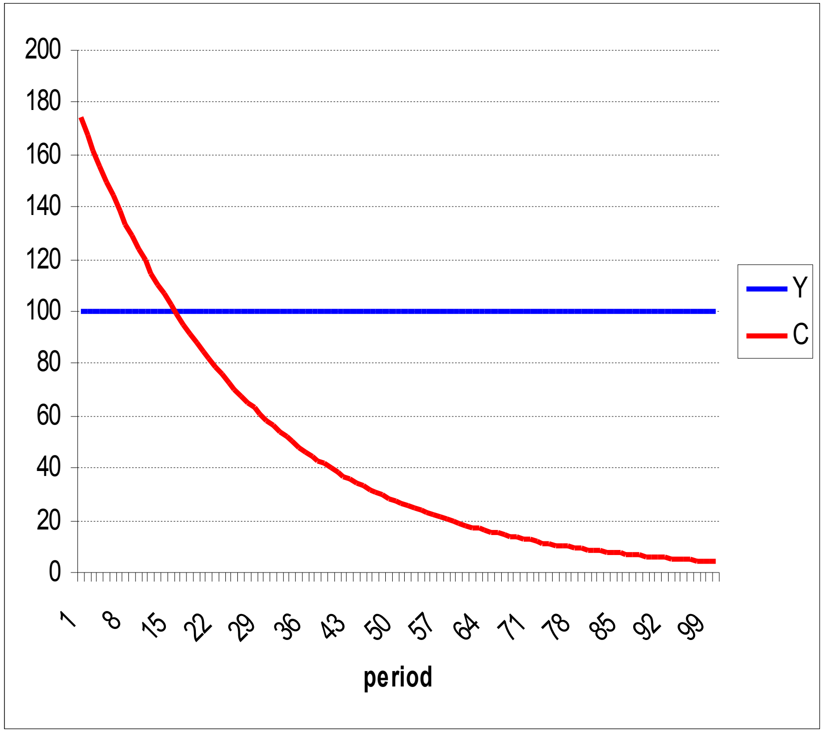
- Assumption of perfect capital mobility;
- Technology is assumed to be the same everywhere;
- Countries *(i)* may differ in terms of their preference parameters;
- In general, $r^* \neq \rho_i + \sigma_i g$.

$$MaxU_i = L_0 \sum_{t=0}^{\infty} \beta_i^t (1+n)^t u(C_t); \qquad \beta_i = \frac{1}{1+\rho_i}$$

$$u_i(c_t) = \frac{c_t^{1-\sigma_i}}{1-\sigma_i}$$

Example – Impatient Country

$$Y = 100;\ \ g,n = 0; I,G = 0;\ \ \beta_i = \frac{1}{1.07};\ r^* = 0.05;\ \sigma_i = \frac{1}{2};\ \ B_1 = 0$$



- Example with no population and technology growth;
- Consumption of impatient people is falling over time towards zero;
- In general, consumption in intensive form is falling towards zero.

Open Ec.: Equilibrium

- In equilibrium, the world interest rate is given by the most patient economy;
- This economy accumulates all the world‘s assets;
- All the other countries accumulate huge debt and their consumption in intensive form declines towards zero in the limit.
- Absurd result (but look at the accumulation of assets in Asia vs. debt in the US).

$$\rho_1 + \sigma_1 g < \rho_2 + \sigma_2 g < \rho_3 + \sigma_3 g < \ldots < \rho_n + \sigma_n g$$

$$r^* = \rho_1 + \sigma_1 g$$

Open Economy: Speed of Convergence

- Marginal product of capital equalizes world-wide; $f_k' = r^*$
- Unless there are differences in technology, convergence in infinitely fast;
- This is, of course, out of line with reality;
- This suggests that the international financial markets have some imperfections. Possible solutions:
  - OLG model (death as a nice borrowing constraint ☺);
  - Risk premium increasing with the debt level (simple ad hoc shortcut)
  - Human capital + borrowing constraints.

Open Ec. with Human Capital

- Two types of capital: physical (K) and human (H);
- Households can borrow only against physical capital, not human capital (can not be seized by creditors);
- As a result, human capital has to be accumulated gradually over time.

$$MaxU = L_0 \sum_{t=0}^{\infty} \beta^t (1+n)^t u(C_t); \qquad \beta = \frac{1}{1+\rho} \qquad (23)$$

$$Y_t = F(K_t, H_t, E_t L_t) \qquad (24)$$

$$H_{t+1} - H_t + K_{t+1} - K_t + B_{t+1} - B_t = F(K_t, H_t, E_t L_t) - C_t + r^* B_t \qquad (25)$$

Open Ec. with Human Capital

- In converging economy with high marginal product of capital, people borrow to the extent of capital *K*, i.e. $-B_t = K_t$;
- Assume a simple Cobb-Douglas production function.

$$H_{t+1} - H_t = F\left(K_t, H_t, E_t L_t\right) - C_t - r^* K_t \qquad (26)$$

$$h_{t+1} - h_t = \frac{\chi\left(h_t\right)^{\gamma} - c_t - r^* k_t}{1+z} - \frac{z}{1+z} h_t \qquad (27)$$

$$Y_t = K_t^{\alpha} H_t^{\phi} \left(E_t L_t\right)^{1-\alpha-\phi} \qquad (28)$$

$$y_t = k_t^{\alpha} h_t^{\phi} \qquad (29)$$

Open Ec. with Human Capital

$$f'_{k,t} = \alpha k_t^{\alpha-1} h_t^{\phi} = r^* \qquad (30,31)$$

$$r^* = \alpha k_t^{-1} k_t^{\alpha} h_t^{\phi} = \alpha \frac{y_t}{k_t} \qquad (32,33)$$

$$k_t = \alpha \frac{y_t}{r^*} \qquad (34)$$

$$y_t = \left(\alpha \frac{y_t}{r^*} \right)^{\alpha} h_t^{\phi} \qquad (35)$$

- Physical capital is invested to equalize its marginal product with the world real interest rate *r*;*
- Reduced-form, intensive production function:

Open Ec. with Human Capital

$$y_t = \left(\frac{\alpha}{r^*} \right)^{\frac{\alpha}{1-\alpha}} h_t^{\frac{\phi}{1-\alpha}} \qquad (38)$$

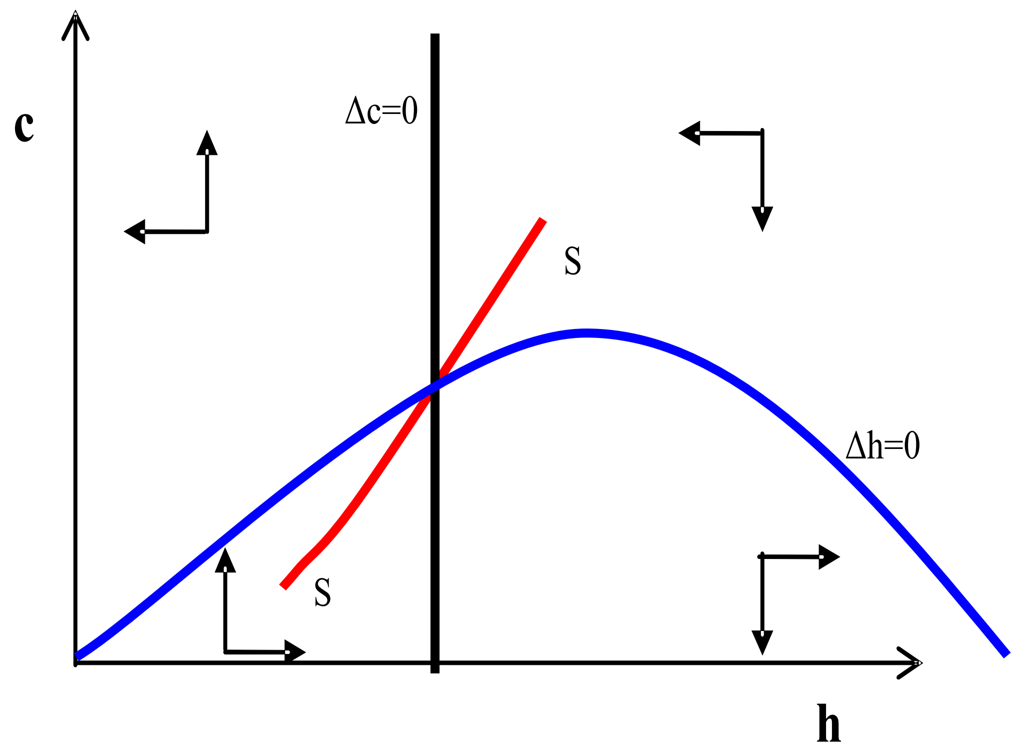
$$y_t = \chi h_t^{\nu} \quad ; \quad \chi \equiv \left(\frac{\alpha}{r^*} \right)^{\frac{\alpha}{1-\alpha}} ; \nu \equiv \frac{\phi}{1-\alpha} \qquad (39)$$

$$\frac{c_{t+1}}{c_t} = \frac{\beta^{\frac{1}{\sigma}} \left[1 + \nu (1-\alpha) \chi h_t^{\nu - 1} \right]^{\frac{1}{\sigma}}}{1+g} \qquad (44)$$

$$h_{t+1} - h_t = \frac{(1-\alpha)\chi h_t^{\nu - 1} - c_t}{1+z} - \frac{z}{1+z} h_t \qquad (45)$$

- Results look similar (except of the (1-α) term) to the closed-economy growth model, with *k* replaced by *h.*

Phase Diagram with Human Capital



- Convergence to the steady state along the saddle path (SS) due to gradual accumulation of human capital;
- Convergence speed depends on $v=\phi/(1-\alpha)$;
- It is faster than in a closed economy with both types of capital, but not infinitely fast.

Summary

- The open economy growth model leads to weird results if perfect capital mobility is assumed (all wealth accumulated by one country; infinitely fast convergence);
- Some form of a borrowing constraint is needed to bring the model in line with reality;
- The model with human capital and borrowing restricted to the physical capital leads to nice results, similar to the closed-economy growth model;
- Access to the global financial markets does increase the speed of convergence, but not to infinity.