L6. Simple monetary model of the exchange rate and Dornbusch’s ‘overshooting’ model

Jarek Hurník

www.jaromir-hurnik.wbs.cz

Simple monetary model

(1) $m_t - p_t = -\eta\, i_{t+1} + \phi\, y_t$ ... Money demand

(2) $p_{\mathrm{t}} \equiv e_t + p_t^*$ ... PPP

(3) $\mathrm{E}_{\mathrm{t}} e_{t+1} - e_t = i_{t+1} - i_{t+1}^*$ ... UIP

(4) $m_t = \mu\, m_{t-1} + \varepsilon_t$ ... Policy rule

Simple monetary model

Substitution of (2) and (3) in money demand yields

$$e_t = \eta\,(\mathrm{E}_t e_{t+1} - e_t) + m_t - \phi y_t - p_t^* + \eta\, i_{t+1}^*$$

Which is a stochastic rational expectations first order difference equation that can be solved as

$$e_t = \frac{1}{1+\eta} \sum_{t=0}^{\infty} \left(\frac{\eta}{1+\eta} \right)^t E_t (m_t - \phi y_t - p_t^* + \eta\, i_{t+1}^*)$$

Simple monetary model

For a reduced-form solution one needs to add the policy rule

$$m_t = \mu m_{t-1} + \varepsilon_t \qquad \mu \geq 1$$

Assuming y, p^*, i^* constant and normalized to zero, the reduced-form solution is

$$e_t = m_t + \frac{\eta}{1 + \mu\eta} m_t$$

Dornbusch 'overshooting' model

(1) $i_{t+1} = i^*_{t+1} + e_{t+1} - e_t$ ... UIP

(2) $m_t - p_t = -\eta\, i_{t+1} + \phi y_t$ ... Money demand

(3) $y^d_t = \bar{y} + \delta\left(e_t + p^* - p_t - \bar{q}\right)$ ... Aggregate demand

(4) $q_t \equiv e_t + p^* - p_t$ ... Definition of real ER

(5) $p_{t+1} - p_t = \psi\left(y^d_t - \bar{y}\right) + \left(\tilde{p}_{t+1} - \tilde{p}_t\right)$ ... Phillips curve

$\tilde{p}_t = e_t + p^*_t - \bar{q}_t$ ... Market clearing price

$1 > \phi\,\delta$ ... Assumption (condition for ER over-shooting)

Dornbusch 'overshooting' model

$$\tilde{p}_t = e_t + p_t^* - \bar{q}_t = \tilde{p}_t = e_t + p^* - \bar{q}$$

$$\tilde{p}_{t+1} = e_{t+1} + p_{t+1}^* - \bar{q}_{t+1} = e_{t+1} + p^* - \bar{q}$$

$$\tilde{p}_{t+1} - \tilde{p}_t = e_{t+1} - e_t$$

$$p_{t+1} - p_t = \psi\left(y_t^d - \bar{y}\right) + \left(e_{t+1} - e_t\right)$$

$$y_t^d - \bar{y} = \delta\left(q_t - \bar{q}\right)$$

$$p_{t+1} - p_t = \psi\delta\left(q_t - \bar{q}\right) + \left(e_{t+1} - e_t\right)$$

$$q_{t+1} - q_t = -\psi\delta\left(q_t - \bar{q}\right)$$

$$m_t - p_t = -\eta\, i_{t+1} + \phi y_t$$

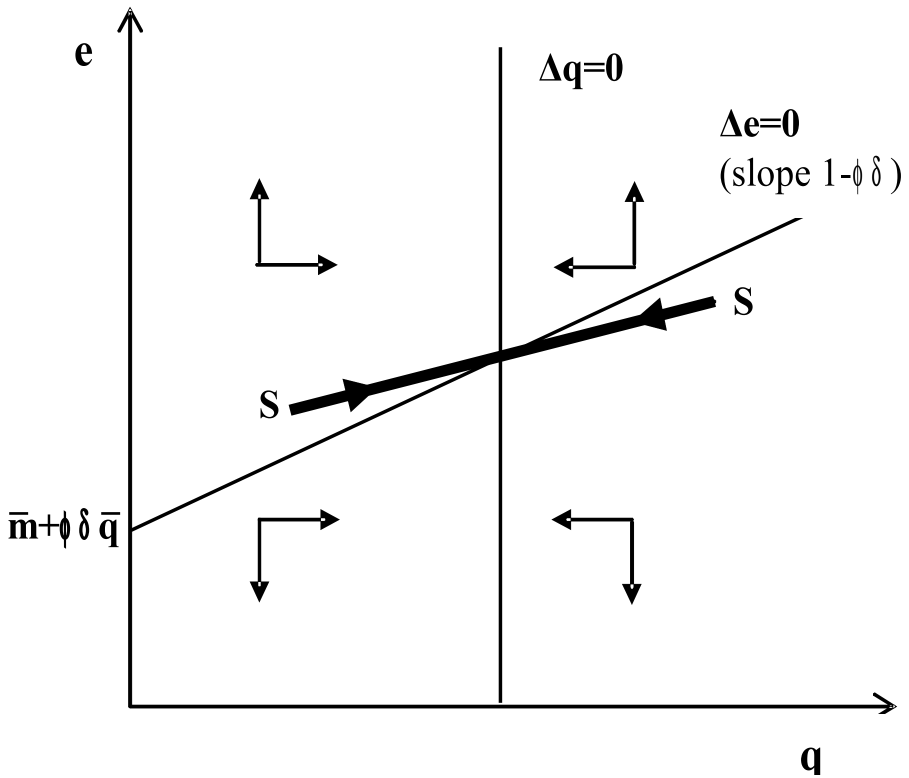
$$i_{t+1} = i^* + e_{t+1} - e_t \qquad y_t = \bar{y} + \delta\left(e_t + p^* - p_t - \bar{q}\right)$$

$$(ass.\ p^* = \bar{y} = i^* = 0)$$

$$m_t - e_t + q_t = -\eta\left(e_{t+1} - e_t\right) + \phi\delta\left(q_t - \bar{q}\right)$$

$$e_{t+1} - e_t = \frac{e_t}{\eta} - \frac{1 - \phi\delta}{\eta} q_t - \frac{m_t + \phi\delta\bar{q}}{\eta}$$

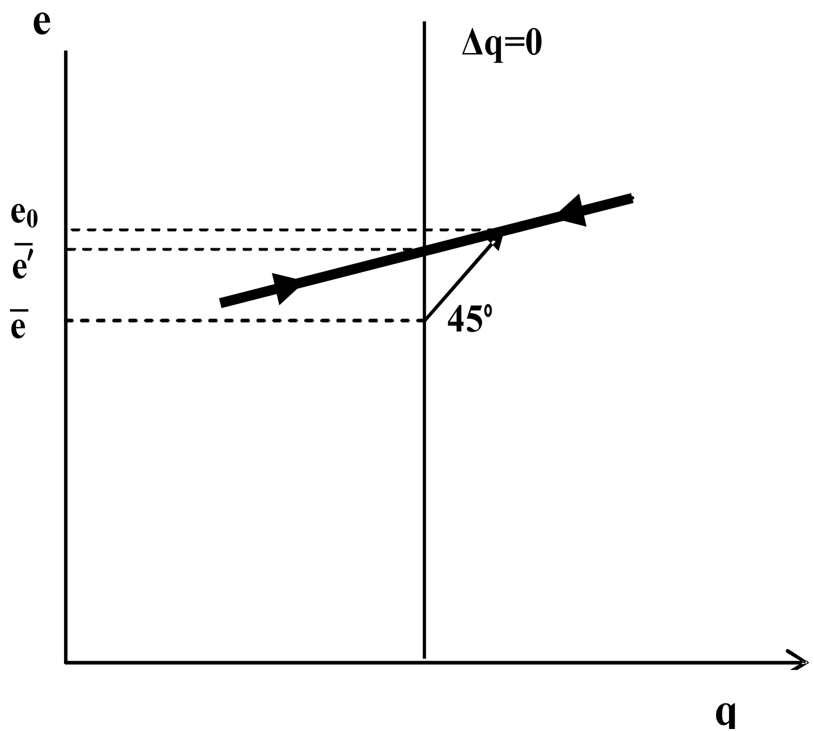
Dornbusch 'overshooting' model



$$\Delta q_{t+1} = -\psi\,\delta\left(q_t - \overline{q}\right)$$

$$\Delta e_{t+1} = \frac{1}{\eta}\left[e_t - \left(1-\phi\,\delta\right)q_t - \left(\phi\,\delta\,\overline{q} + m_t\right)\right]$$

Permanent increase in money stock



- After an unexpected one-off increase in money supply, the nominal ER initially depreciates more than proportionately;
- Intuition: if the depreciation was proportionate, output would rise by $\delta\Delta m^s$ and money demand by $\phi\delta\Delta m^s<\Delta m^s \Rightarrow$ IR must fall.